AIEEE 2012

Test Booklet Code - C

Questions and Solutions

PAPER - 1 : MATHEMATICS, PHYSICS & CHEMISTRY

PART- A : MATHEMATICS

1. The equation $e^{-\sin x} - e^{-\sin x} - 4 = 0$ has :
(1) infinite number of real roots
(2) no real roots
(3) exactly one real root
(4) exactly four real roots

1. (2)
$e^{\sin x} - e^{-\sin x} - 4 = 0$
Let $e^{\sin x} = t$
$t - \frac{1}{t} - 4 = 0$
$\Rightarrow t^2 - 4t - 1 = 0$
$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2}$
$\Rightarrow t = 2 \pm \sqrt{5}$
$e^{\sin x} = 2 + \sqrt{5}$ Not possible [2.7 < e < 2.8]
$e^{\sin x} = 2 - \sqrt{5}$ Not possible [Never Negative]

2. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is :
(1) $\frac{\pi}{6}$
(2) $\frac{\pi}{2}$
(3) $\frac{\pi}{3}$
(4) $\frac{\pi}{4}$

2. (3)
$\vec{c}\cdot\vec{d} = 0$
$(\hat{a}+2b)\cdot(5\hat{a}-4b) = 0$
$5+6\hat{a}\cdot b-8 = 0$
$\Rightarrow \hat{a}\cdot b = \frac{1}{2}$
$\Rightarrow |\hat{a}||b| \cos\theta = \frac{1}{2}$
$\Rightarrow \cos\theta = \frac{\pi}{3}$

3. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is :
(1) 9/7
(2) 7/9
(3) 2/9
(4) 9/2

3. (3)

$\frac{4}{3}\pi R^3 = 4500\pi$

$\Rightarrow\ R^3 = 3 \times 1125 \qquad \Rightarrow\ R = 15\text{m}$

$\frac{dv}{dt} = \frac{4}{3}\times 3\pi r^2 \frac{dr}{dt} = 72\pi$

$\Rightarrow\ \frac{dr}{dt} = \frac{18}{r^2} \qquad \ldots (1)$

Also, $4500\pi - 72\pi \times 49 = \frac{4}{3}\pi r^3$

$\Rightarrow\ r = 9\text{m}$

$\therefore\ \frac{dr}{dt} = \frac{18}{81} = \frac{2}{9}$ m/minute

4. **Statement 1:** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + … + (361 + 380 + 400) is 8000.

Statement 2: $\sum_{k=1}^{n} \left(k^3 - (k-1)^3\right) = n^3$, for any natural number n.

(1) Statement 1 is false, Statement 2 is true.
(2) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.
(3) Statement 1 is true, Statement 2 is true; Statement 2 is **not** a correct explanation for statement 1.
(4) Statement 1 is true, Statement 2 is false.

4. (2)

Statement 2:

$\sum_{K=1}^{n} K^3 - (K-1)^3$

$\Rightarrow\ \left(\frac{n(n+1)}{2}\right)^2 - \left(\frac{n(n-1)}{2}\right)^3 = n^3$

Statement '2' is correct

Statement 1:

$(1^3-0^3) + (2^3-1^3) + (3^3-2^3) + \ldots (20^3-19)^3 = 8000$

Statement '1' is correct

and statement '2' explain statement '1'

5. The negation of the statement

"If I become a teacher, then I will open a school", is :

(1) I will become a teacher and I will not open a school.
(2) Either I will not become a teacher or I will not open a school.
(3) Neither I will become a teacher nor I will open a school.
(4) I will not become a teacher or I will open a school.

5. (1)

6. If the integral $\int \frac{5\tan x}{\tan x - 2}dx = x + a\,\ell n\left|\sin x - 2\cos x\right| + k$ then a is equal to :

(1) −1 (2) −2 (3) 1 (4) 2

6. (4)

$$\int \frac{5\tan x}{\tan x - 2}dx = x + a\, ln\, |\sin x - 2\cos x| + K$$

Differentiating on both side

$$\Rightarrow \quad \frac{5\tan x}{\tan x - 2} = 1 + \frac{a[\cos x + 2\sin x]}{\sin x - 2\cos}$$

$$\Rightarrow \quad \frac{5\sin x}{\sin x - 2\cos x} = \frac{\sin x - 2\cos x + a(\cos x + 2\sin x)}{\sin x - 2\cos x}$$

Equating co-efficient of both side

$$\underbrace{5 = 1 + 2a}_{\sin x},\ \underbrace{0 = -2 + a}_{\cos x}$$

$\Rightarrow$ a = 2

7. **Statement 1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement 2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, ($m \neq 0$) is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(1) Statement 1 is false, Statement 2 is true.
(2) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(3) Statement 1 is true, Statement 2 is true, Statement 2 is **not** a correct explanation for Statement 1.
(4) Statement 1 is true, Statement 2 is false.

7. (2)

Put $y = mx + \frac{4\sqrt{3}}{m}$ in $2x^2 + y^2 = 4$

$$\Rightarrow 2x^2 + \left(mx + \frac{4\sqrt{3}}{m}\right)^2 = 4 \qquad \Rightarrow (2 + m^2)x^2 + 8\sqrt{3}\,x + \left(\frac{48}{m^2} - 4\right) = 0$$

$\because$ $y = mx + \frac{4\sqrt{3}}{m}$ is a tangent, discriminant of the above quadratic equation must be zero.

$$\therefore \quad (8\sqrt{3})^2 = 4(2 + m^2)\left(\frac{48}{m^2} - 4\right)$$

$\Rightarrow m^4 + 2m^2 - 24 = 0 \qquad \Rightarrow (m^2 + 6)(m^2 - 4) = 0$

$\Rightarrow$ $m = \pm 2$

$\therefore$ Statement (2) is a correct explanation of statement (1).

8. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to :

(1) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (2) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (3) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (4) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

8. (4)

$$AU_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a \\ 2a+b \\ 3a+2b+c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ a = 1; b = –2; c = 1

$$U_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$AU_2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ 2x+y \\ 3x+2y+z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\Rightarrow$ x = 0; y = 1; z = –2

$$U_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

$$U_1 + U_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

9. If n is a positive integer, then $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$ is :

(1) an irrational number
(2) an odd positive integer
(3) an even positive integer
(4) a rational number other than positive integers

9. (1)

$$(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$$

$$2\left[2n_{C_1}(\sqrt{3})^{2n-1} + 2n_{C_3}(\sqrt{3})^{2n-3} + \ldots\ldots 2n_{C_{2n-1}}(\sqrt{3})\right]$$

which is Irrational Number

10. If 100 times the 100th term of an AP with non zero common difference equals the 50 times its 50th term, then the 150th term of this AP is :

(1) –150
(2) 150 times its 50th term
(3) 150
(4) zero

10. (4)
Let first term 'a' and common difference 'd'
100 [a + 99 d] = 50 [a + 49 d]
$\Rightarrow$ a = –149 d
Now,
T_{150} = a + 149 d
$\Rightarrow$ $T_{150} = 0$

11. In a ΔPQR, if 3 sin P + 4 cos Q = 6 and 4 sin Q + 3 cos P = 1, then the angle R is equal to :
(1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

11. (2)
$P + Q + R = \pi$
$3 \sin P + 4 \cos Q = 6$ .... (i)
$4 \sin Q + 3 \cos P = 1$ .... (ii)
squaring and adding (i) and (ii)
$9 + 16 + 24 \sin (P + Q) = 37$
$\Rightarrow \sin (P + Q) = \frac{1}{2}$
$\Rightarrow P + Q = \frac{\pi}{6}; \frac{5\pi}{6}$
if $P + Q = \frac{\pi}{6}$
$\Rightarrow 0 < \sin P < \frac{1}{2}$ and $\frac{\sqrt{3}}{2} < \cos Q < 1$
while violate statement 3 sin P + 4 cos Q = 6

12. An equation of a plane parallel to the plane x – 2y + 2z – 5 = 0 and at a unit distance from the origin is :
(1) x – 2y + 2z – 3 = 0 (2) x – 2y + 2z + 1 = 0
(3) x – 2y + 2z – 1 = 0 (4) x – 2y + 2z + 5 = 0

12. (1)
Equation of plane parallel to the plane
$x - 2y + 2z - 5 = 0$
is $x - 2y + 2z + t = 0$
Distance from (0, 0) is
$\Rightarrow \left|\frac{t}{3}\right| = 1$ $\Rightarrow t = 3$, or -3
$\Rightarrow x - 2y + 2z - 3 = 0$

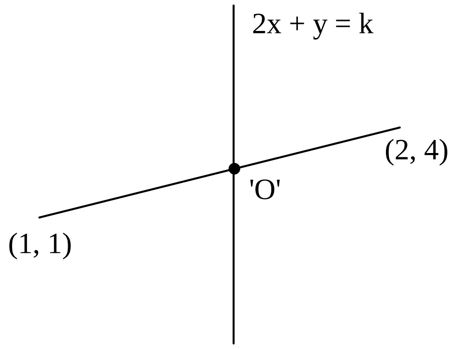
13. If the line 2x + y = k passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals :
(1) 29/5 (2) 5 (3) 6 (4) 11/5

13. (3)

Co–ordinate of pt 'O'

$\Rightarrow \dfrac{3\times2+2\times1}{5}, \dfrac{3\times4+2\times1}{5}$

$\Rightarrow \left(\dfrac{8}{5}, \dfrac{14}{5}\right)$



Satisfying the equation 2x + y = k

$2\left(\dfrac{8}{5}\right)+\left(\dfrac{14}{5}\right) = k$

$\dfrac{30}{5} = k \Rightarrow k = 6$

14. Let $x_1, x_2,\ldots,x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be their variance.

Statement 1 : Variance of $2x_1, 2x_2,\ldots,2x_n$ is $4\sigma^2$.

Statement 2 : Arithmetic mean of $2x_1, 2x_2,\ldots,2x_n$ is $4\bar{x}$.

(1) Statement 1 is false, Statement 2 is true.
(2) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(3) Statement 1 is true, Statement 2 is true, Statement 2 is **not** a correct explanation for Statement 1.
(4) Statement 1 is true, Statement 2 is false.

14. (4)

Statement 2 is false because Arthmetic mean of

$\dfrac{2x_1+2x_2+2x_3+\ldots+2x_n}{n}$

$2\bar{x}$.

15. The population p(t) at time t of a certain mouse species satisfies the differential equation $\dfrac{dp(t)}{dt} = 0.5\,p(t) - 450$. If p(0) = 850, then the time at which the population becomes zero is :

(1) $2\,\ell n18$ (2) $\ell n9$ (3) $\dfrac{1}{2}\,\ell n18$ (4) $\ell n18$

15. (1)

$\dfrac{d\,P(t)}{dt} - 0.5\,P(t) = -450$

I.F $= e^{\int -0.5\,dt} \Rightarrow e^{\frac{-t}{2}}$

$P(t)\cdot e^{\frac{-t}{2}} = \int -450\cdot e^{\frac{-t}{2}}\,dt$

$P(t)\,e^{\frac{-t}{2}} = 900\,e^{\frac{-t}{2}} + c$

At t = 0

850 = 900 + c $\Rightarrow$ c = –50

$P(t)\,e^{\frac{-t}{2}} = 900\,e^{\frac{-t}{2}} - 50$

The time of which P(t) = 0 is

$$0 = 900\,e^{\frac{-t}{2}} - 50$$

$$\Rightarrow \quad e^{\frac{-t}{2}} = \frac{1}{18}$$

$$\Rightarrow \quad -\frac{t}{2} = -\frac{\ell n}{18}$$

$$\Rightarrow \quad t = \frac{2\ell n}{18}$$

16. Let a, b $\in$ R be such that the function f given by f(x) = $\ell n|x| + bx^2 + ax,\ x \neq 0$ has extreme values at x = –1 and x = 2.

Statement 1 : f has local maximum at x = –1 and at x = 2.

Statement 2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$.

(1) Statement 1 is false, Statement 2 is true.
(2) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(3) Statement 1 is true, Statement 2 is true, Statement 2 is **not** a correct explanation for Statement 1.
(4) Statement 1 is true, Statement 2 is false.

16. (2)

$$f'(x) = \frac{1}{x} + 2bx + a = 0$$

$$\Rightarrow \quad 2bx^2 + ax + 1 = 0$$

$$\therefore \quad -\frac{a}{2b} = -1 + 2 \quad \text{and} \quad \frac{1}{2b} = (-1)(2)$$

$$\Rightarrow \quad a = \frac{1}{2}, \qquad b = \frac{-1}{4}$$

Also, $f''(x) = -\frac{1}{x^2} - \frac{1}{2} < 0\ \forall\ x$

$\therefore$ x = –1 and 2 are points of maxima.

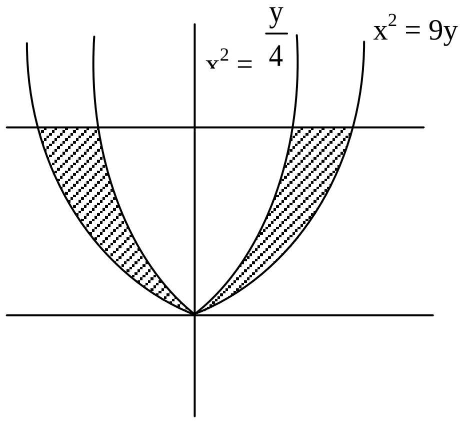
17. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line y = 2 is :

(1) $20\sqrt{2}$ (2) $\frac{10\sqrt{2}}{3}$ (3) $\frac{20\sqrt{2}}{3}$ (4) $10\sqrt{2}$

17. (3)

$$A = 2\left(\int_0^2 \sqrt{9y}\,dy - \int_0^2 \sqrt{\frac{y}{4}}\,dy\right)$$

$$= 2\int_0^2 \frac{5}{2}\sqrt{y}\,dy = \frac{20\sqrt{2}}{3}$$



18. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is :

(1) 880 (2) 629 (3) 630 (4) 879

18. (4)

No. of ways = (10 + 1) (9 + 1) (7 + 1) – 1 = 879

19. If f : R $\to$ R is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is :

(1) continuous for every real x.

(2) discontinuous only at x = 0.

(3) discontinuous only at non-zero integral values of x.

(4) continuous only at x = 0.

19. (1)

$f(x) = [x]\cos\left(\pi x - \frac{\pi}{2}\right) = [x]\sin(\pi x)$

which is continuous for all x

At n $\in$ I,

$\lim\limits_{x\to n}[x]\sin(\pi x) = [n]\sin(\pi n) = 0$

20. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to :

(1) –1 (2) $\frac{2}{9}$ (3) $\frac{9}{2}$ (4) 0

20. (3)

If two lines r = $\vec{a}_1 + t\vec{b}_1$ and r = $\vec{a}_2 + t\vec{b}_2$ intersect, then $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = 0$

$\Rightarrow \left[\vec{a}_2 - \vec{a}_1 \quad \vec{b}_1 \quad \vec{b}_2\right] = 0$

$$\Rightarrow \begin{vmatrix} 2 & 2 & 1 \\ K+1 & 3 & 2 \\ -1 & 4 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ 2(–5) – 2(K + 3) + 1(4K + 4 + 3) = 0

$\Rightarrow$ –10 – 2K – 6 + 4K + 7 = 0

$\Rightarrow K = \frac{9}{2}$

21. Three numbers are chosen at random without replacement from {1, 2, 3, …, 8}. The probability that their minimum is 3, given that their maximum is 6, is :

(1) 3/8 (2) 1/5 (3) 1/4 (4) 2/5

21. (2)

Let x = Minimum is 3

y = Maximum is 6

$\therefore \quad P\left(\frac{x}{y}\right) = \frac{2}{{}^5C_2} = \frac{1}{5}$

22. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies:

(1) either on the real axis or on a circle passing through the origin.
(2) on a circle with centre at the origin.
(3) either on the real axis or on a circle not passing through the origin.
(4) on the imaginary axis.

22. (1)

Let $z = x + iy$

Then $\frac{(x+iy)^2}{(x+iy)-1} = K, \quad K \in R$

$\Rightarrow (x^2 - y^2 - Kx + K) + i(2x - K)y = 0$

$\Rightarrow x^2 - y^2 - Kx + K = 0 \quad \ldots (1)$

and $y(2x - K) = 0 \quad \ldots (2)$

$\therefore y = 0$ or $x^2 - 2x + y^2 = 0$

$\Rightarrow y = 0$ or $(x - 1)^2 + y^2 = 1$

23. Let P and Q be 3×3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to :

(1) -2 (2) 1 (3) 0 (4) -1

23. (3)

$P^3 = Q^3 \quad \ldots$ (i)

$Q^2P = P^2Q \quad \ldots$ (ii)

(i) + (ii)

$P^3 + Q^2P = Q^3 + P^2Q$

$P(P^2 + Q^2) = Q(Q^2 + P^2)$

$(P - Q)(P^2 + Q^2) = 0$

$\Rightarrow \det.(P^2 + Q^2) = 0 \quad (\because P - Q \neq 0)$

24. If $g(x) = \int_0^x \cos 4t\, dt$, then $g(x + \pi)$ equals :

(1) $\frac{g(x)}{g(\pi)}$ (2) $g(x) + g(\pi)$ (3) $g(x) - g(\pi)$ (4) $g(x) . g(\pi)$

24. (2), (3)

$$g(x + \pi) = \int_0^{x+\pi} \cos(4t)dt = \int_0^x \cos(4t)dt + \int_x^{x+\pi} \cos(4t)dt$$

$$= g(x) + \int_0^{\pi} \cos(4t)dt = g(x) + g(\pi)$$

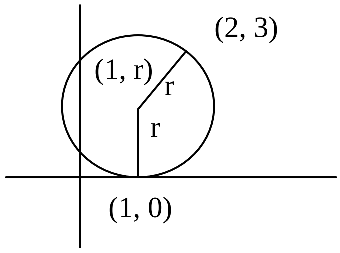
But since $g(\pi) = 0$, $g(x) - g(\pi)$ is also a correct option.

25. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is :

(1) 10/3 (2) 3/5 (3) 6/5 (4) 5/3

25. (1)

If circle touches x–axis at (1, 0)
then centre of the circle will be (1, r),
where r is radius of the circle



$\therefore\ (2-1)^2 + (3-r)^2 = r^2$

$= \quad 1 + 9 - 6r = 0 \Rightarrow r = \dfrac{10}{6}$

$\therefore$ Diameter = $\dfrac{10}{3}$

26. Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is :

(1) 5^2 (2) 3^5 (3) 2^5 (4) 5^3

26. (2)

Number of different ordered pairs = $3 \times 3 \times 3 \times 3 \times 3 = 3^5$

($\because$ Every element has 3 options, either it can be in y but not in z, or it can be in z but not in y or in neither of y or z)

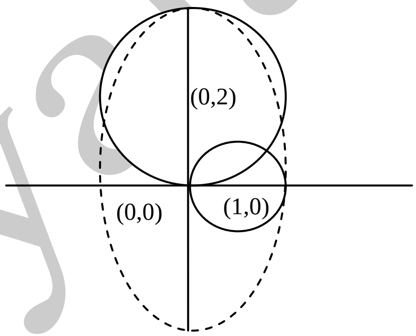
27. An ellipse is drawn by taking a diameter of the circle $(x-1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y-2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axis, then the equation of the ellipse is :

(1) $4x^2 + y^2 = 4$ (2) $x^2 + 4y^2 = 8$
(3) $4x^2 + y^2 = 8$ (4) $x^2 + 4y^2 = 16$

27. (none of these)

Equation of ellipse is $\dfrac{x^2}{2^2} + \dfrac{y^2}{4^2} = 1$

$\Rightarrow\ 4x^2 + y^2 = 16$



28. Consider the function, $f(x) = |x-2| + |x-5|$, $x \in R$.

Statement 1 : $f'(4) = 0$

Statement 2 : f is continuous in [2, 5], differentiable in (2, 5) and f(2) = f(5).

(1) Statement 1 is false, statement 2 is true.
(2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.
(3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.
(4) Statement 1 is true, statement 2 is false.

28. (3)

By Rolle's theorem, Statement 2 is true.

But by Rolle's theorem, we can not conclude that $f'(4) = 0$

$f'(4) = 0$ because f(x) is a constant function in [2, 5].

29. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is :

(1) $-1/4$ (2) -4 (3) -2 (4) $-1/2$

29. (3)

Let the line passing through (1, 2) be $\frac{x}{a}+\frac{y}{b}=1$

$\therefore\ \frac{1}{a}+\frac{2}{b}=1 \Rightarrow a = \frac{b}{b-2}$

Area of triangle, A = $\frac{1}{2}ab=\frac{1}{2}\cdot\frac{b}{(b-2)}\cdot b$

$$\frac{dA}{db}=\frac{1}{2}\left[\frac{(b-2)2b-b^2\cdot 1}{(b-2)^2}\right] = 0 \Rightarrow b = 0, 4$$

$\therefore$ b = 4 and a = 2;
Hence slope = -2

30. Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q}, \overrightarrow{AD}=\vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by

(1) $\vec{r}=3\vec{q}-\frac{3\,\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\vec{p}$ (2) $\vec{r}=-\vec{q}+\left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

(3) $\vec{r}=\vec{q}-\left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ (4) $\vec{r}=-3\vec{q}+\frac{3\,\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\vec{p}$

30. (2)

$\vec{r}\cdot\vec{p}=0$

and $\vec{q}+\vec{r}=\lambda\,\vec{p}$

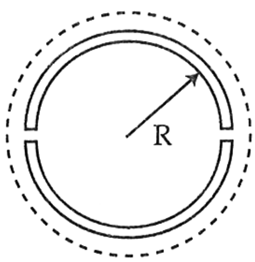
$\Rightarrow \vec{q}\cdot\vec{p}+\vec{r}\cdot\vec{p}=\lambda\,\vec{p}\cdot\vec{p} \Rightarrow \lambda = \frac{\vec{q}\cdot\vec{p}}{\vec{p}\cdot\vec{p}}$

So, $\vec{r}=\lambda\,\vec{p}-\vec{q}$

$\vec{r}=-\vec{q}+\left(\frac{\vec{q}\cdot\vec{p}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

PART- B : PHYSICS

31. A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is :



(1) $2\pi SY\alpha\Delta T$ (2) $SY\alpha\Delta T$ (3) $\pi SY\alpha\Delta T$ (4) $2SY\alpha\Delta T$

31. (4)

$\Delta\ell = \alpha L . \Delta T$

Strain $= \dfrac{\Delta\ell}{\ell} = \alpha\,\Delta T$

Stress $=$ Y . strain $= Y\,\alpha\,.\,\Delta T$

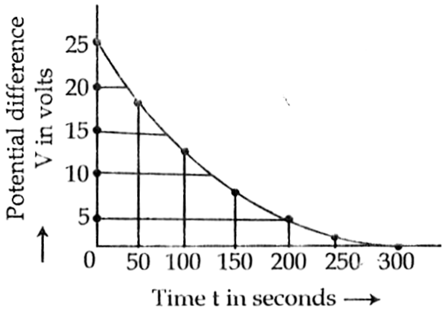
$\dfrac{F}{S} = Y\alpha\Delta T$

$F = S\,Y\,\alpha\,\Delta T$

Force will be exerted on both side.

$\therefore$ Force $2SY\alpha\Delta T$

32. The figure shows an experimental plot for discharging of a capacitor in an R−C circuit. The time constant τ of this circuit lies between :

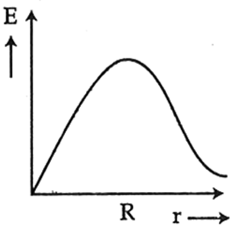


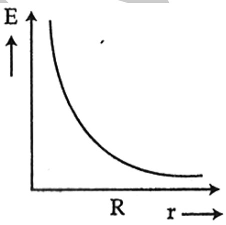
(1) 150 sec and 200 sec
(2) 0 and 50 sec
(3) 50 sec and 100 sec
(4) 100 sec and 150 sec

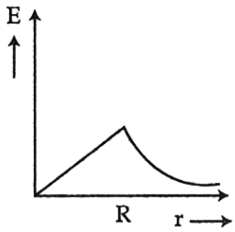
32. (4)

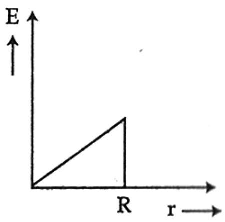
In an time constant potential will drop to 37% of maximum voltage
i.e. will became $0.37 \times 25 = 9.25$ V
From graph it is clear, it will happen between 100s to 150s.

33. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be

(1)


(2)


(3)


(4)


33. (3)

Inside the sphere $E \propto r$ (r = radial distance from the centre)

Outside the sphere $E \propto \dfrac{1}{r^2}$

34. An electromagnetic wave in vacuum has the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then :

(1) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B}\times\vec{E}$
(2) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E}\times\vec{B}$
(3) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E}\times\vec{B}$
(4) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B}\times\vec{E}$

34. (2)

$\vec{X} || \vec{E}$ and $\vec{K} || \ \vec{E} \times \vec{B}$

35. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = 0s to t – τs, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds :

(1) $\frac{0.693}{b}$ (2) b (3) $\frac{1}{b}$ (4) $\frac{2}{b}$

35. (4)

The equation of motion is that of a damped harmonic oscillator :

$\ddot{\theta} \;=\; -\omega^2 \theta - b\,\dot{\theta}$. Solution of this equation is of the form :

$\theta \;=\; \theta_0\, e^{-\frac{bt}{2}} \sin(\omega t' + \delta)$.

Thus : amplitude becomes $\left(\frac{\theta_0}{e}\right)$ after a time t = $\frac{2}{b}$

36. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be :

(1) 2 (2) 3 (3) 5 (4) 6

36. (4)

No. of spectral lines = $\frac{n(n-1)}{2}$, here n = 4

37. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to :

(1) development of air current when the plate is placed.
(2) induction of electrical charge on the plate
(3) shielding of magnetic lines of force as aluminium is a paramagnetic material
(4) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping

37. (4)

Eddy current are induced in aluminum magnetic field due to which opposes the oscillation of the coil.

38. The mass of spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be :

(1) 6.4×10^{11} Joules (2) 6.4×10^{8} Joules
(3) 6.4×10^{9} Joules (4) 6.4×10^{10} Joules

38. (4)

Required energy $= \frac{1}{2} m V_e^2$

where $V_e = \sqrt{2g}\,R$

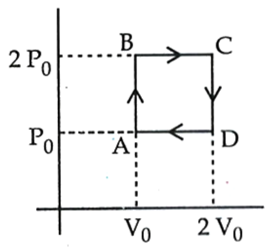
Required energy $= \frac{1}{2} m \times 2gR$

$= mgR$

$= 1000 \times 10 \times 6400 \times 1000 = 6.4 \times 10^{10}$ J

39. Helium gas goes through a cycle ABCDA (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly : (Assume the gas to be close to ideal gas)

(1) 15.4% (2) 9.1%
(3) 10.5% (4) 12.5%



39. (1)

$$\eta = \frac{W}{Q_{in}}$$

W = Area under the curve
$= P_0V_0$

$$Q_{AB} = nC_V\,\Delta T \quad = \frac{f}{2}nR\,\Delta T \quad = \frac{3}{2}P_0V_0$$

$$Q_{BC} = nC_P\,\Delta T \quad = \frac{f+2}{2}nR\,\Delta T \quad = \frac{5}{2}(2P_0V_0) \qquad = 5\,P_0V_0$$

$Q_{CD} < 0$
$Q_{DA} < 0$

$$Q_{in} = \left(\frac{3}{2}+5\right)P_0V_0 \;=\; \frac{13}{2}P_0V_0$$

$$\eta = \frac{P_0V_0}{\frac{13}{2}P_0V_0} \qquad = \frac{2}{13}$$

$\eta(\%) = 15.4\%$

40. In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by :

(1) $\frac{I_m}{9}(4+5\cos\phi)$ (2) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$

(3) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$ (4) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$

40. (4)

$I \;\alpha\; A^2$

$I_1 = KA^2 = I_0$ (say)

$I_2 = K(2A)^2 = 4I_0$

$\therefore\; I = I_1 + I_2 + 2\sqrt{I_1\,I_2}\,\cos\phi$

$= I_0 + 4I_0 + (2)\,(2I_0)\cos\phi$

$= 5I_0 + 4I_0\cos\phi$

$= I_0\,[1 + 4(1 + \cos\phi)]$

$I = I_0\,[1 + 8\cos^2\phi/2]$ ….(1)

And,

$I_m = I_0 + 4I_0 + 2(2I_0)\cos 0$

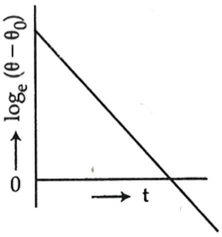
$I_m = 9\,I_0$

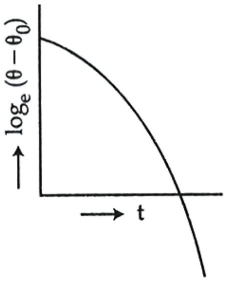
$\therefore \quad I_0 \quad = \quad \dfrac{I_m}{9}$

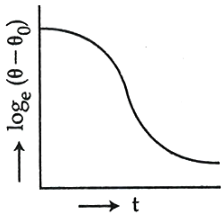
from equation (1),

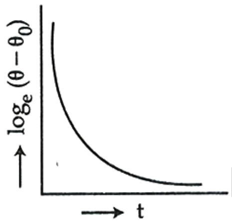
$\therefore \quad I_0 \quad = \quad \dfrac{I_m}{9}[1 + 8\cos^2 \phi/2]$

41. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is :

(1) 

(2) 

(3) 

(4) 

41. (1)

Newton's Law of Cooling,

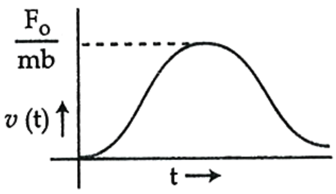
$$-\frac{dv}{dt} = -K(\theta - \theta_0)$$

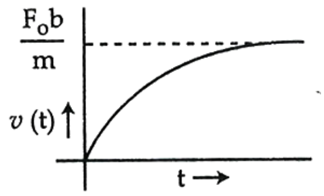
$$\int \frac{dv}{\theta - \theta_0} = -K\int dt$$

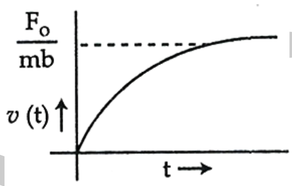
$$\ell n(\theta - \theta_0) = Kt + C$$

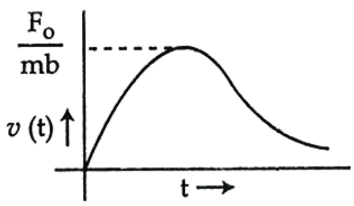
$\therefore$ It represents equation of straight line in $\ell n(\theta - \theta_0)$ Vs t plot.

42. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force $F(t) = F_0e^{-bt}$ in the x direction. Its speed v(t) is depicted by which of the following curves?

(1) 

(2) 

(3) 

(4) 

42. (3)

$$m\frac{dv}{dt} = F_0\, e^{-bt}$$

$$\int_0^v dv = \frac{F_0}{m}\int_0^t e^{-bt}\, dt$$

$$v = -\frac{F_0}{mb}\left[e^{-bt}\right]_0^t$$

$$= -\frac{F_0}{mb}\left[e^{-bt} - 1\right]$$

$$v = -\frac{F_0}{mb}e^{-bt} + \frac{F_0}{mb}$$

at $t = 0$, $v = 0$

at $t \to \infty$, $v \to \frac{F_0}{mb}$

43. Two electric bulbs marked 25 W – 220 V and 100 W – 220 V are connected in series to a 440 V supply. Which of the bulbs will fuse?

(1) both (2) 100 W (3) 25 W (4) neither

43. (3)

$P \propto \frac{1}{R}$

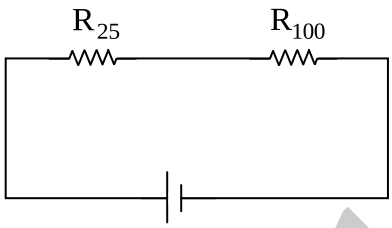
$$\frac{R_{25}}{R_{100}} = \frac{100}{25}$$

$$R_{25} = 4R_{100}$$

$$V_{25} = 440\,\frac{R_{25}}{R_{25} + R_{100}}$$

$$= 440 \times \frac{4}{5}$$

$$= 352 \text{ V}$$



Which is more than 220 V.

$\therefore$ 25 W bulb will fuse.

44. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is :

(1) 6% (2) zero (3) 1% (4) 3%

44. (1)

$$R = \frac{V}{I}$$

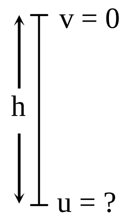
$$\frac{\Delta R}{R}\times 100 = \frac{\Delta V}{V}\times 100 + \frac{4\,I}{I}\times 100 = 3 + 3 = 6\%.$$

45. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be :

(1) $20\sqrt{2}$ m (2) 10 m (3) $10\sqrt{2}$ m (4) 20 m

45. (4)

According to boy's own capability, lets assume that the maximum speed with which he can throw a ball is ‘u’, then



$$v^2 = u^2 + 2as$$

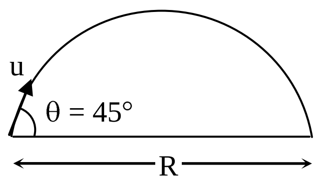
$$0 = u^2 - 2gh$$

$$u^2 = 2gh = 2 \times 10 \times 10 = 200 \text{ m/s}^2$$

Now, for maximum horizontal range

$\theta = 45°$

$$\left(R_{max} = \frac{u^2}{g} = \frac{200}{10} = 20\,\text{m}\right)$$



46. *This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.*

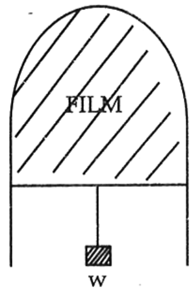
Statement 1 : Davisson – Germer experiment established the wave nature of electrons.

Statement 2 : If electrons have wave nature, they can interfere and show diffraction.

(1) Statement 1 is false, Statement 2 is true.
(2) Statement 1 is true, Statement 2 is false
(3) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1
(4) Statement 1 is true, Statement 2 is true, Statement 2 is **not** the correct explanation of Statement 1

46. (3)

47. A thin liquid film formed between a U–shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is :

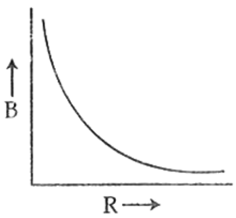


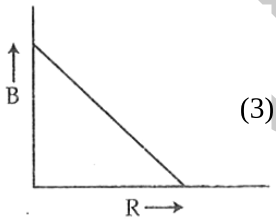
(1) 0.0125 Nm^{-1} (2) 0.1 Nm^{-1}
(3) 0.05 Nm^{-1} (D) 0.025 Nm^{-1}

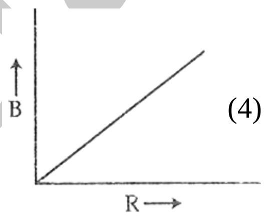
47. (4)

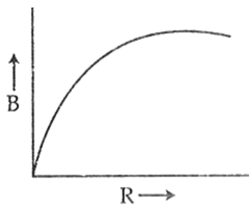
Surface tension = $\dfrac{mg}{2D} = \dfrac{1.5\times10^{-2}}{2\times30\times10^{-2}} = 0.025$ N/m

48. A charge Q is uniformly distributed over the surface of non–conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure :

(1) 

(2) 

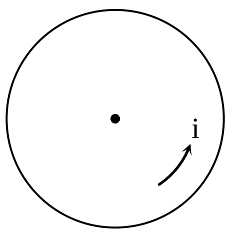
(3) 

(4) 

48. (1)

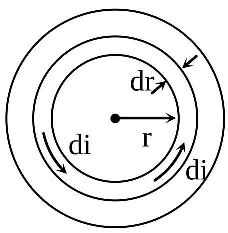
$$\frac{i}{\pi R^2} = \frac{di}{2\pi r\,dr}$$

$$di = \frac{2i\,r\,dr}{R^2}$$



$$dB = \frac{\mu_0(di)}{2r}$$

$$= \frac{\mu_0}{2r}\,\frac{2ir\,dr}{R^2}$$

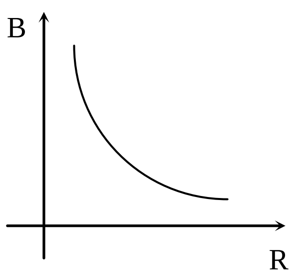


$$dB = \frac{\mu_0 i}{R^2}dr$$

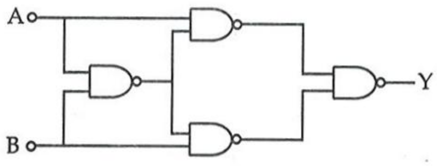
$$B = \frac{\mu_0 i}{R^2}\int_0^R dr = \frac{\mu_0 i}{R^2}(R-0)$$

$$B = \frac{\mu_0 i}{R}$$

$$B \propto \frac{1}{R}$$



49. Truth table for system of four NAND gates as shown in figure is :



(1)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(3)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

49. (1)

GATE 1 : $\overline{A\cdot B}$

GATE 2 : $\overline{A\cdot\overline{A\cdot B}}$

GATE 3 : $\overline{B\cdot\overline{A\cdot B}}$

$$Y = \overline{\overline{(A\cdot\overline{A\cdot B})}\cdot\overline{(B\cdot\overline{A\cdot B})}} \qquad \{\text{using } \bar{A}\cdot\bar{B} = \bar{A}+\bar{B} \text{ and } \overline{A+B} = \bar{A}\cdot\bar{B}\}$$

$$Y = \overline{\overline{(A\cdot(\bar{A}+\bar{B}))}\cdot\overline{(B\cdot(\bar{A}+\bar{B}))}}$$

$$= \overline{\overline{(A\cdot\bar{A}+A\cdot\bar{B})}\cdot\overline{(B\cdot\bar{A}+B\cdot\bar{B})}} \qquad \{\text{since } A\cdot\bar{A} = B\cdot\bar{B} = 0\}$$

$$\therefore\ Y = \overline{\overline{(A\cdot\bar{B})}\cdot\overline{(\bar{A}\cdot B)}}$$

$$= \overline{(\bar{A}+B)\cdot(A+\bar{B})}$$

$$Y = (A\cdot\bar{B}) + (\bar{A}\cdot B)$$

So,

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

50. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^{-6} m)

(1) 80 km (2) 16 km (3) 40 km (4) 64 km

50. (1)

Maximum distance d = $\sqrt{2Rh}$

$\therefore$ d $= \sqrt{2\times 6.4\times 10^{6}\times 500}$

$= \sqrt{64\times 10^{8}}$

$= 8 \times 10^{4}$m

$\therefore$ d = 80 km

51. Assume that a neutron breaks into a proton and an electron. The energy released during this process is :

(Mass of neutron = 1.6725×10^{-27} kg

Mass of proton = 1.6725×10^{-27} kg

Mass of electron = 9×10^{-31} kg)

(1) 0.73 MeV (2) 7.10 MeV (3) 6.30 MeV (4) 5.4 MeV

51. (1)

$$\Delta m = (m_e + m_p) - (m_0)$$
$$m_e = 9.1093897 \times 10^{-31}\text{ kg}$$
$$m_p = 1.6726231 \times 10^{-27}\text{ kg}$$
$$m_n = 1.6749286 \times 10^{-27}\text{ kg}$$
$$\Delta m = 1.395 \times 10^{-30}\text{ kg}$$

$$E = (\Delta m)\,C^2$$
$$= 1.395 \times 10^{-30} \times (3 \times 10^8)^2$$
$$= 1.255 \times 10^{-13}\text{ Joules}$$
$$= \frac{1.255\times 10^{-13}}{1.6\times 10^{-19}}\text{eV}$$
$$E = 0.78\text{ MeV}$$

52. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be :
(1) efficiency of Carnot engine cannot be made larger than 50%
(2) 1200 K
(3) 750 K
(4) 600 K

52. (3)
Efficiency of Carnot engine operating between T_C and T_M is given by

$$\eta = 1 - \frac{T_C}{T_M}$$

In first case : $\eta = 0.4$: $T_M = 500$ K

$$\therefore\ 0.4 = 1 - \frac{T_C}{500} \Rightarrow T_C = 0.6 \times 500 = 300\text{ K}$$

For a desired efficiency of 0.6 : $T_C = 300$ K

$$0.6 = 1 - \frac{300}{T_M} \Rightarrow \frac{300}{T_M} = 0.4$$

$$T_M = \frac{3000}{4} = 750\,\text{K}$$

53. *This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.*
It two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.
Statement 1 : If stretched by the same amount, work done on S_1, will be more than that on S_2
Statement 2 : $k_1 < k_2$
(1) Statement 1 is false, Statement 2 is true.
(2) Statement 1 is true, Statement 2 is false
(3) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1
(4) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

53. (1)
Since the applied force is same and extension is also occurring to statement–1; work done on both going is of same. Hence statement 1 cannot be true.

54. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is :
(1) $m_1 r_1 : m_2 r_2$ (2) $m_1 : m_2$ (3) $r_1 : r_2$ (4) 1 : 1

54. (3)

$$\frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2} \Rightarrow \frac{r_1}{r_2} = \frac{v_1}{v_2}$$

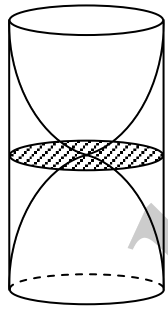
$$\frac{a_1}{a_2} = \frac{v_1^2}{r_1} \cdot \frac{r_2}{v_2^2} = \left(\frac{r_2}{r_1}\right) \cdot \frac{r_1^2}{r_2^2} = \frac{r_1}{r_2}$$

55. A cylindrical tube, open at both ends, has a fundamental frequency f, in air. The tube is dipped vertically in water so that half of it is water. The fundamental frequency of the air−column is now:

(1) f (2) $\frac{f}{2}$ (3) $\frac{3f}{4}$ (4) 2f

55. (1)

Fundamental frequency remains constant same.



56. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film?

(1) 7.2 m (2) 2.4 m (3) 3.2 m (4) 5.6 m

56. [4]

In case without glass plate :

u = –2.4 m v = +0.12 m

$$\frac{1}{0.12} - \frac{1}{(-2.4)} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{100}{12} + \frac{10}{24} = \frac{210}{24} = \frac{35}{4}\left(\frac{1}{\mu}\right)$$

Introducing the glass plate ;

The shift in position of image due to plate = $t\left(1 - \frac{1}{\mu}\right)$

$$\frac{1}{3}\text{cm} = 1 \times \left(1 - \frac{2}{3}\right)$$

Thus the image would be formed at $\left(12 - \frac{1}{3}\right) = \frac{35}{3}$ cm from lens had the plate been absent.

$$\frac{3}{35} - \frac{1}{v} = \frac{35}{400} \Rightarrow \frac{1}{v} = \frac{3}{35} - \frac{35}{400} = \frac{3}{35} - \frac{7}{80}$$

$$\therefore \frac{1}{v} = \frac{240 - 245}{35 \times 80}$$

$$v = \frac{35 \times 80}{5} = -560 \text{ cm}$$

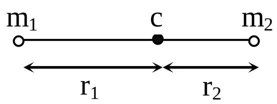
$$= -5.6 \text{ m.}$$

57. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by : (n is an integer)

(1) $\dfrac{(m_1+m_2)^2 n^2h^2}{2m_1^2m_2^2r^2}$ (2) $\dfrac{n^2h^2}{2(m_1+m_2)r^2}$ (3) $\dfrac{2n^2h^2}{(m_1+m_2)r^2}$ (4) $\dfrac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$

57. (4)

$$m_1r_1 = m_2r_2 \quad \Rightarrow \quad r_2 = r_1\frac{m_1}{m_2}$$



$$r_1 + r_2 = r$$

$$\therefore \quad r_1 = \left(\frac{m_2}{m_1+m_2}\right)r \;\Rightarrow\; r_1 = \left(\frac{\mu}{m_1}\right)r\,;\;\; r_2 = \left(\frac{\mu}{m_2}\right)r^2$$

$$\therefore \quad I \quad = \quad m_1r_1^2 + m_2r_2^2 \quad = \quad r^2\mu^2\left[\frac{1}{m_1}+\frac{1}{m_2}\right]$$

$$\Rightarrow I \quad = \quad \mu r^2$$

Rotational Energy $= \dfrac{L^2}{2I} = \dfrac{m^2k^2}{2\mu r^2}$

58. A spectrometer gives the following reading when used to measure the angle of a prism.
Main scale reading : 58.5 degree
Vernier scale reading : 09 divisions
Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data :

(1) 58.59 degree (2) 58.77 degree (3) 58.65 degree (4) 59 degree

58. (3)

Least count = $\left(\dfrac{0.5}{30}\right)^{o}$

$$\therefore \quad \text{Correct Angle} \quad = \quad 58.5^{o} + 9\times\frac{0.5}{30}$$

$$= \quad 58.5^{o} + \frac{3}{20} = 58.5^{o} + 0.15 = 58.65$$

59. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.
An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point out side the sphere. The electric potential at infinity is zero.

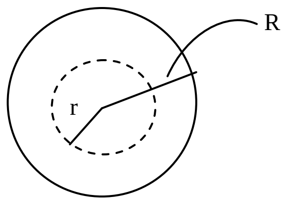
Statement 1 : When a charge 'q' is taken from the centre to the surface of the sphere, its potential energy changes by $\dfrac{qp}{3\varepsilon_0}$.

Statement 2 : The electric field at a distance r (r < R) from the centre of the sphere is $\dfrac{\rho r}{3\varepsilon_0}$

(1) Statement 1 is true, Statement 2 is true; Statement 2 is not the correct explanation of Statement 1.
(2) Statement 1 is true Statement 2 is false.
(3) Statement 1 is false Statement 2 is true
(4) Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation of Statement 1

59. (3)

Using gauss' law :



$$E \cdot 4\pi r^2 = \frac{p}{\varepsilon_0} \cdot \frac{4}{3}\pi r^3$$

$$\Rightarrow \; E = \frac{pr}{3\varepsilon_0}$$

$\therefore$ Statement 2 is correct. Statement 1 is dimensionally incorrect.

60. Proton, Deuteron and alpha particle of the same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relations is correct?

(1) $r_\alpha = r_p = r_d$ (2) $r_\alpha = r_p < r_d$ (3) $r_\alpha > r_d > r_p$ (4) $r_\alpha = r_d > r_p$

60. (2)

$$r = \frac{mV}{qB}$$

$$\frac{1}{2}mv^2 = K$$

$$V = \sqrt{\frac{2K}{m}}$$

$$r = \frac{\sqrt{2mK}}{qB}$$

$$r \propto \frac{\sqrt{m}}{q}$$

$$r_P \propto \frac{\sqrt{m}}{q} \; ; \; r_D \propto \frac{\sqrt{2m}}{q} \; ; \; r_\alpha \propto \frac{\sqrt{4\,m}}{2q}$$

$$\therefore \; r_P = r_\alpha < r_D$$

PART- C : CHEMISTRY

61. Which among the following will be named as dibromidobis (ethylene diamine) chromium (III) bromide ?

(1) $[Cr(en)_3]Br_3$ (2) $[Cr(en)_2Br_2]Br$ (3) $[Cr(en)Br_4]^-$ (4) $[Cr(en)Br_2]Br$

61. (2)

Bis (ethylene diamine) means two ethylene diamine ligands; dibromido means 2 Br^- Ligands. Only option 2 fits this description.

62. Which method of purification is represented by the following equation :

$$Ti_{(s)} + 2I_{2(g)} \xrightarrow{523\,K} Ti\,I_{4(g)} \xrightarrow{1700\,K} Ti_{(s)} + 2I_{2(g)}$$

(1) Zone refining (2) Cupellation (3) Poling (4) Van Arkel

62. (4)

The reaction sequence given in the problem is the Van Arkel method of refining/purification of Ti metal.

63. Lithium forms body centred cubic structure. The length of the side of its unit cell is 351 pm. Atomic radius of the lithium will be :

(1) 75 pm (2) 300 pm (3) 240 pm (4) 152 pm

63. (4)

For B.C.C.

$4r = \sqrt{3}a$

$r = \dfrac{\sqrt{3}\times a}{4} = \dfrac{\sqrt{3}\times 351}{4} = \dfrac{1.732\times 351}{4}$ = 152 pm

64. The molecule having smallest bond angle is :

(1) NCl_3 (2) $AsCl_3$ (3) $SbCl_3$ (4) PCl_3

64. (3)

The smallest bond angle is in $SbCl_3$

65. Which of the following compounds can be detected by Molisch's test ?

(1) Nitro compounds (2) Sugars (3) Amines (4) Primary alcohols

65. (2)

Sugars are determined by Molisch's test

66. The incorrect expression among the following is :

(1) $\dfrac{\Delta G_{system}}{\Delta S_{total}} = -T$

(2) In isothermal process, $w_{reversible} = -nRT \ln \dfrac{V_f}{V_i}$

(3) $\ln K = \dfrac{\Delta H^\circ - T\Delta S^\circ}{RT}$

(4) $K = e^{-\Delta G^\circ/RT}$

66. (3)

→ $\Delta G_{sys} = -T\Delta S_{Total}$ is true

→ In isothermal process, $W_{rev} = -nRT\, \ell n \dfrac{v_1}{v_2}$ is correct

→ $RT\,\ell n\, K = -\Delta G^\circ = T\Delta S^\circ - \Delta H^\circ$ which is given incorrect

→ $k = e^{-\Delta G^\circ/RT}$ is a correct formula

67. The density of a solution prepared by dissolving 120 g of urea (mol. mass = 60 u) in 1000 g of water is 1.15 g/mL. The molarity of this solution is :

(1) 0.50 M (2) 1.78 M (3) 1.02 M (4) 2.05 M

67. (4)

$$\text{Molarity M} = \frac{n_{solution}}{V_{solution}}$$

$$= \frac{W_s}{M_s \times V_{Solution}} = \frac{W_s \times d_{solution}}{M_s \times W_{solution}} = \frac{120\times 1.15\times 1000}{60\times 1120} = 2.05\text{ M}$$

68. The species which can best serve as an initiator for the cationic polymerization is :

(1) $LiAlH_4$ (2) HNO_3 (3) $AlCl_3$ (4) BuLi

68. (3)

Cationic polymerization requires acid catalyst.

69. Which of the following on thermal–decomposition yields a basic as well as an acidic oxide ?
(1) $NaNO_3$ (2) $KClO_3$ (3) $CaCO_3$ (4) NH_4NO_3

69. (3)

$CaCO_3$ on decomposition gives CaO which is basic and CO_2 which is acidic.

70. The standard reduction potentials for Zn^{2+} / Zn, Ni^{2+} / Ni , and Fe^{2+} / Fe are –0.76, –0.23 and –0.44 V respectively. The reaction $X + Y^{2+} \rightarrow X^{2+} + Y$ will be spontaneous when :
(1) X = Ni, Y = Fe (2) X = Ni, Y = Zn (3) X = Fe, Y = Zn (4) X = Zn, Y = Ni

70. (4)

1) $Ni + Fe^{+2} \rightarrow Ni^{2+} + Fe$

$\Rightarrow E_{cell} = E^{\circ}_{cathode} - E^{\circ}_{anode}$

$= E^{\circ}_{Fe^{+2}/Fe} - E^{\circ}_{Ni^{+2}/Ni}$

$= - 0.44 - (0.23)$

$= - 0.44 + 0.23 < 0$; so non spontaneous

2) $Ni + Zn^{+2} \rightarrow Ni^{2+} + Zn$

$\Rightarrow E_{cell} = E^{\circ}_{Zn^{2+}/Zn} - E^{\circ}_{Ni^{2+}/Ni}$

$= - 0.76 - (- 0.23)$

$= - 0.73 + 0.23 < 0$; so non spontaneous

Therefore (4) is spontaneous.

Which in $Zn + Ni^{2+} \rightarrow Ni + Zn^{2+}$

71 According to Freundlich adsorption isotherm, which of the following is correct ?

(1) $\frac{x}{m} \propto p^0$

(2) $\frac{x}{m} \propto p^1$

(3) $\frac{x}{m} \propto p^{1/n}$

(4) All the above are correct for different ranges of pressure.

71. (4)

$\frac{x}{m} \propto p^{\frac{1}{n}}$

At n = 1 ; $\frac{x}{m} \propto p$

n = ∞ ; $\frac{x}{m} \propto p^{\circ}$

at n ; $\frac{x}{m} \propto p^{\frac{1}{n}}$.

72. The equilibrium constant (K_c) for the reaction $N_{2(g)} + O_{2(g)} \rightarrow 2NO_{(g)}$ at temperature T is 4×10^{-4}. The value of K_c for the reaction, $NO_{(g)} \rightarrow \frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)}$ at the same temperature is :

(1) 0.02 (2) 2.5×10^{-2} (3) 4×10^{-4} (4) 50.0

72. (4)

$$N_{2\,(g)} + O_{2\,(g)} \xrightarrow{K_C} 2\,NO \qquad : K_C$$

$$2NO \xrightarrow{\frac{1}{K_C}} N_2 + O_2 \qquad : K'_C = \frac{1}{K_C}$$

$$\Rightarrow \quad NO \longrightarrow \frac{1}{2}N_2 + \frac{1}{2}O_2 \qquad : K''_C = \frac{1}{\sqrt{K_C}}$$

$$\Rightarrow \quad K''_C = \frac{1}{\sqrt{4\times10^{-4}}}$$

$$= \frac{1}{2}\times100$$

$$= 50$$

73. The compressibility factor for a real gas at high pressure is :

(1) 1 + RT / pb (2) 1 (3) 1 + pb / RT (4) 1 – pb / RT

73. (3)

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT \qquad \Rightarrow \text{at high pressure, } \frac{an^2}{V^2} \rightarrow 0$$

$$\Rightarrow \quad PV = nRT + Pnb$$

$$Z = 1 + \frac{Pb}{RT}$$

74. Which one of the following statements is correct ?

(1) All amino acids except lysine are optically active.
(2) All amino acids are optically active.
(3) All amino acids except glycine are optically active
(4) All amino acids except glutamic acid are optically active.

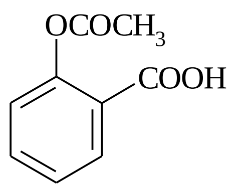
74. (3)

$$\underset{|}{NH_2}$$
$CH_2 - COOH$ (glycine) which does not contain chiral carbon.

75. Aspirin is known as :

(1) Acetyl salicylic acid (2) Phenyl salicylate
(3) Acetyl salicylate (4) Methyl salicylic acid

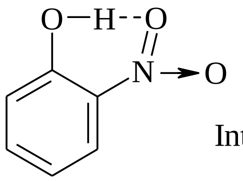
75. (1)



; acetyl salicylic acid (Aspirin)

76. Ortho – Nitrophenol is less soluble in water than p – and m – Nitrophenols because :
(1) o – Nitrophenol is more volatile in steam than those of m – and p – isomers
(2) o – Nitrophenol shows Intramolecular H – bonding
(3) o – Nitrophenol shows Intermolecular H – bonding
(4) Melting point of o – Nitropehnol is lower than those of m – and p – isomers.

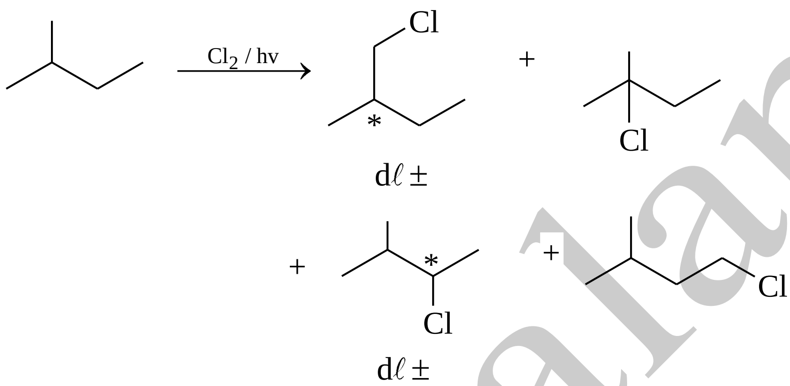
76. (2)



Intramolcular hydrogen bonding

77. How many chiral compounds are possible on monochlorination of 2 – methyl butane ?
(1) 8 (2) 2 (3) 4 (4) 6

77. (3)



78. Very pure hydrogen (99.9 %) can be made by which of the following processes ?
(1) Reaction of methane with steam
(2) Mixing natural hydrocarbons of high molecular weight
(3) Electrolysis of water
(4) Reaction of salt like hydrides with water

78. (3)
Very pure (99.9 %) hydrogen is made by electrolysis of water.

79. The electrons identified by quantum numbers n and 1 :
(a) n = 4, l = 1 (b) n = 4, l = 0 (c) n = 3, l = 2 (d) n = 3, l = 1
can be placed in order of increasing energy as :
(1) (c) < (d) < (b) < (a) (2) (d) < (b) < (c) < (a)
(3) (b) < (d) < (a) < (c) (4) (a) < (c) < (b) < (d)

79. (2)

Energy of an electron in an orbital depends on $(n + \ell)$ sum. Lower the $(n + \ell)$ value lower is the energy and if $(n + \ell)$ value is same, lower n means lower energy. Applying these rules we get $E_d < E_b < E_c < E_a$. Thus option 2 is correct.

80. For a first order reaction, (A) $\rightarrow$ products, the concentration of A changes from 0.1 M to 0.025 M in 40 minutes. The rate of reaction when the concentration of A is 0.01 M , is :
(1) 1.73×10^{-5} M / min (2) 3.47×10^{-4} M / min
(3) 3.47×10^{-5} M / min (4) 1.73×10^{-4} M / min

80. (2)

$$k = \frac{2.303}{t}\log\left(\frac{a_0}{a_0 - x}\right)$$

$$= \frac{2.303}{40}\times\log\left(\frac{0.1}{0.025}\right) = \frac{2.303}{40}\times\log 4$$

$$= \frac{2.303}{40}\times 2\log 2 = \frac{0.693\times 2}{40} = \frac{0.693}{20}$$

$$-\frac{dx}{dt} = k(A) \text{ where } A = (A_0 - x)$$

$$= \frac{0.693}{20}\times 0.01$$

$$= 3.47\times 10^{-4}\text{ M / min.}$$

81. Iron exhibits +2 and +3 oxidation states. Which of the following statements about iron is incorrect ?

(1) Ferrous oxide is more basic in nature than the ferric oxide.
(2) Ferrous compounds are relatively more ionic than the corresponding ferric compounds.
(3) Ferrous compounds are less volatile than the corresponding ferric compounds.
(4) Ferrous compounds are more easily hydrolysed than the corresponding ferric compounds.

81. (4)

Fe^{3+} is a harder acid as compared to Fe^{2+}. Thus it would prefer OH^- as a ligand compared to water. Thus, Fe^{3+} compounds are more easily hydrolyzed. Option (4) is correct.

82. The pH of a 0.1 molar solution of the acid HQ is 3. The value of the ionization constant, Ka of this acid is :

(1) 3×10^{-1} (2) 1×10^{-3} (3) 1×10^{-5} (4) 1×10^{-7}

82. (3)

$$HQ \rightleftharpoons H^+ + Q^-$$

$$K_a = C_0\alpha^2$$

$$= \frac{(C_0\alpha)^2}{C_0} = \frac{[H^+]^2}{C_0}$$

$$pH = 3 \Rightarrow [H^+] = 10^{-3}$$

$$K_a = \frac{[10^{-3}]^2}{0.1} = 10^{-5}$$

83. Which branched chain isomer of the hydrocarbon with molecular mass 72u gives only one isomer of mono substituted alkyl halide ?

(1) Tertiary butyl chloride (2) Neopentane
(3) Isohexane (4) Neohexane

83. (2)

$14n + 2 = 72 \Rightarrow n = 5 \Rightarrow C_5H_{12}$

$$H_3C-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH_3 \xrightarrow{Cl_2/hr} H_3C-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH_2-Cl$$

84. K_f for water is 1.86 K kg mol^{-1}. If your automobile radiator holds 1.0 kg of water, how many grams of ethylene glycol ($C_2H_6O_2$) must you add to get the freezing point of the solution lowered to −2.8°C ?
(1) 72 g (2) 93 g (3) 39 g (4) 27 g

84. (2)

$$\Delta T^{\circ}_f = k_f \times m = k_f \times \frac{W_s}{M_s \times W_{Solution}}$$

M.W. ($C_2H_6O_2$) = 24 + 6 + 32 = 62

$$\omega_s = \frac{\Delta T^{\circ}_f \times M_s \times W_{Solution}}{k_f} = \frac{2.8 \times 62 \times 1}{1.86} = 93.39$$

85. What is DDT among the following :
(1) Greenhouse gas (2) A fertilizer
(3) Biodegradable pollutant (4) Non – biodegradable pollutant

85. (4)
Factual

86. The increasing order of the ionic radii of the given isoelectronic species is :
(1) Cl^- , Ca^{2+} , K^+ , S^{2-} (2) S^{2-}, Cl^-, Ca^{2+} , K^+
(3) Ca^{2+} , K^+, Cl^- , S^{2-} (4) K^+ , S^{2-} , Ca^{2+} , Cl^-

86. (3)

For isoelectronic species lower the atomic number, the higher ionic radius. Thus increasing order of size is $Ca^{2+} < K^+ < Cl^- < S^{2-}$. Option 3 is correct.

87. 2 – Hexyne gives trans – 2 – Hexene on treatment with :
(1) Pt / H_2 (2) Li / NH_3 (3) Pd / $BaSO_4$ (4) Li AlH_4

87. (2)

$$H_3C-C\equiv C-CH_2-CH_2-CH_3 \xrightarrow{Li/NH_3} CH_3-\overset{\overset{\displaystyle H}{|}}{C}=\underset{\underset{\displaystyle H}{|}}{C}-CH_2-CH_2-CH_3$$

(trans.)

88. Iodoform can be prepared from all except :
(1) Ethyl methyl ketone (2) Isopropyl alcohol
(3) 3 – Methyl – 2 – butanone (4) Isobutyl alcohol

88. (4)

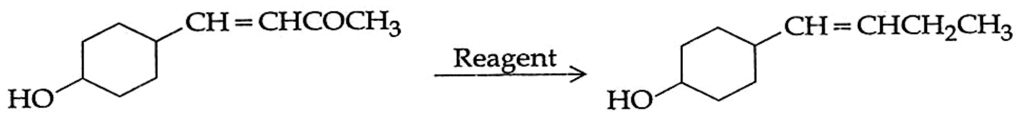
$CH_3-\underset{\underset{\displaystyle CH_3}{|}}{CH}-CH_2-OH$; Isobutyl alcohol does not give iodoform test

89. In which of the following pairs the two species are not isostructural ?
(1) CO_3^{2-} and NO_3^- (2) PCl_4^+ and $SiCl_4$
(3) PF_5 and BrF_5 (4) AlF_6^{3-} and SF_6

89. (3)

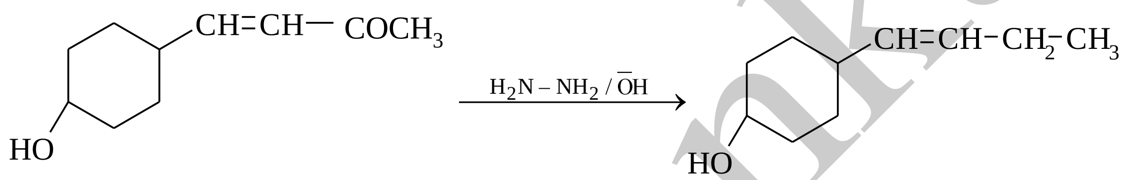
The structure of polyatomic molecule/ion is determined by the number of valence electron pairs around the central atom. In PF_5 valence electron pairs are 5 while in BrF_5 the number is 6. Hence, they are not isostructural. Option 3 is correct.

90. In the given transformation, which of the following is the most appropriate reagent ?



(1) $NH_2\ NH_2\ , \overset{\ominus}{O}H$ (2) Zn – Hg / HCl
(3) Na, Liq . NH_3 (4) $NaBH_4$

90. (1)



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