Guidelines for
Mathematics Laboratory in Schools

Class X

Central Board of Secondary Education
Preet Vihar, Delhi – 110092.

1. Introduction

Taking into consideration the national aspirations and expectations reflected in the recommendations of the National Curriculum Framework developed by NCERT, the Central Board of Secondary Education had initiated a number of steps to make teaching and learning of mathematics at school stage activity-based and experimentation oriented. In addition to issuing directions to its affiliated schools to take necessary action in this regard, a document on '*Mathematics Laboratory in Schools – towards joyful learning*' was brought out by the Board and made available to all the schools. The document primarily aimed at sensitizing the schools and teachers to the concept of Mathematics Laboratory and creating awareness among schools as to how the introduction of Mathematics Laboratory will help in enhancing teaching – learning process in the subject from the very beginning of school education. The document also included a number of suggested hands-on activities.

With the objective of strengthening the concept further, the Board brought out another document '*Guidelines for Mathematics Laboratory in Schools – Class IX*' in the year 2005. The document aimed at providing detailed guidelines to schools with regard to the general layout, physical infrastructure, material and human resources, etc for a Mathematics Laboratory. Besides, the document included a list of hands-on activities and projects, detailed procedure to be followed for carrying out these activities and the scheme of evaluation. In the meantime, the Board had issued two circulars to all its schools with regard to establishing Mathematics Laboratory and introduction of the scheme of internal assessment in the subject. Circular number 10 dated March 2, 2005 clarified that the internal assessment of 20% is to be given on the basis of performance of an individual in activity work, project work and continuous assessment. The schools were also informed through the same circular that the scheme would be effective for Class IX from the academic session 2005-2006 and for Class X from the academic session starting April 2006 i.e. from March, 2007 Examination.

2. About the present document

The present document includes details of all Class X activities to be carried out by the students during the full academic session. Description of a few sample projects has also been included. Since Class X is a terminal stage, great care has been taken to ensure that these activities are directly related to the syllabus, can be easily performed and do not require expensive equipment or materials. A detailed evaluation scheme has also been given. A sincere attempt has been made to ensure that the students or teachers are not put to any kind of stress due to time constraint, curriculum load or any other difficulties in carrying out the proposed activity work. For the sake of completeness and reinforcement of the concept, the present document reiterates the general features of a Mathematics Laboratory given in the earlier document for Class IX.

3. Mathematics Laboratory

3.1 What is a Mathematics Laboratory ?

Mathematics Laboratory is a place where students can learn and explore mathematical concepts and verify mathematical facts and theorems through a variety of activities using different materials. These activities may be carried out by the teacher or the students to explore, to learn, to stimulate interest and develop favourable attitude towards mathematics.

3.2 Need and purpose of Mathematics Laboratory

Some of the ways in which a Mathematics Laboratory can contribute to the learning of the subject are:

- It provides an opportunity to students to understand and internalize the basic mathematical concepts through concrete objects and situations.
- It enables the students to verify or discover several geometrical properties and facts using models or by paper cutting and folding techniques.
- It helps the students to build interest and confidence in learning the subject.
- The laboratory provides opportunity to exhibit the relatedness of mathematical concepts with everyday life.
- It provides greater scope for individual participation in the process of learning and becoming autonomous learners.
- It provides scope for greater involvement of both the mind and the hand which facilates cognition.
- The laboratory allows and encourages the students to think, discuss with each other and the teacher and assimilate the concepts in a more effective manner.
- It enables the teacher to demonstrate, explain and reinforce abstract mathematical ideas by using concrete objects, models, charts, graphs, pictures, posters, etc.

3.3 Design and general layout

A suggested design and general layout of laboratory which can accommodate about 32 students at a time is given here. The design is only a suggestion. The schools may change the design and general layout to suit their own requirements.

3.4 Physical infrastructure and materials

It is envisaged that every school will have a Mathematics Laboratory with a general design and layout as indicated with suitable change, if desired, to meet its own requirements. The minimum materials required to be kept in the laboratory may include furniture, all essential equipment, raw materials and other necessary things to carry out the activities included in the document effectively. The quantity of different materials may vary from one school to another depending upon the size of the group.

3.5 Human Resources

It is desirable that a person with minimum qualification of graduation (with mathematics as one of the subjects) and professional qualification of Bachelor in Education be made incharge of the Mathematics Laboratory. He/she is expected to have special skills and interest to carry out practical work in the subject. The concerned mathematics teacher will accompany the class to the laboratory and the two will jointly conduct the desired activities. A laboratory attendant or laboratory assistant with suitable qualification and desired knowledge in the subject can be an added advantage.

3.6 Time Allocation for activities

It is desirable that about 15% - 20% of the total available time for mathematics be devoted to activities. Proper allocation of periods for laboratory activities may be made in the time table. The total available time may be divided judiciously between theory classes and practical work.

4. Scheme of Evaluation (Class X)

4.1 Internal assessment

It is envisaged that 20% weightage will be given to the internal assessment in the subject. The following weightages have been assigned to Board's theory examination and school based internal assessment for Class X examination. The scheme will be effective from March, 2007 examination onwards.

Theory Examination	:	80 marks
Internal Assessment	:	20 marks

These weightages have also been reflected in the Board's document 'Secondary School Curriculum, 2007, volume I.

Internal assessment of 20 marks, based on school-based examination, will have the following break-up:

Year-end assessment of activities	:	10 marks
Assessment of project work	:	05 marks
Continuous assessment	:	05 marks

4.2 Assessment of Activity Work

The year-end assessment of activities and project work will be done during an organized session of an hour and a half with intimation to the Board. The following parameters may be kept in mind for the same:

a) The proposed internal examination may be organized in the month of February as per the convenience of the schools.

b) Every student may be asked to perform two given activities during the allotted time. Special care may be taken in choosing these two activities to ensure that the students are not put to any kind of stress due to time constraint.

c) The students may be divided into small groups of 20-25 as per the convenience of the schools.

d) The assessment may be carried out by a team of two mathematics teachers including the teacher teaching the particular section.

e) The break-up of 10 marks for assessment of a single activity could be as under:

• Statement of the objective of activity	:	01 mark
• Design or approach to the activity	:	02 marks
• Actual conduct of the activity	:	03 marks
• Description/explanation of the procedure	:	03 marks
• Result and conclusion	:	01 mark

The marks for assessment of two activities (10+10) may be added and then reduced to be out of 10.

Every student may be asked to complete the activities given in this document during the academic year and maintain a proper activity record of this work.

The schools would keep a record of the activity notebook and project work of the students' work as well as answer scripts of this examination for verification by the Board, whenever necessary, for a period of six months.

4.3 Evaluation of project work

Every student will be asked to do at least one project, based on the concepts learnt in the classroom. The project should be preferably carried out individually and not in a group. The project may not be mere repetition or extension of the laboratory activities, but should aim at extension of learning to real life situations. Besides, it should also be somewhat open-ended and innovative. The project can be carried out beyond the school working hours. Some sample projects are given in the document but these are only illustrative in nature. The teacher may encourage the students to take up new projects. The weightage of five marks for project work could be further split up as under

Identification and statement of the project	:	01 mark
Design of the project	:	01 mark
Procedure/processes adopted	:	01 mark
Write-up of the project	:	01 mark
Interpretation of result	:	01 mark

4.4 Continuous Assessment

The procedure given below may be followed for awarding marks for continuous assessment in Class X.

a) Reduce the marks of Class IX annual examination to be out of ten marks.
b) Reduce the marks of Class X first terminal examination to be out of ten marks.
c) Add the marks of (a) and (b) above and get the achievement of the student out of twenty marks.
d) Reduce the total in (c) above to the achievement out of five marks

The marks (out of 5) may be added to the score of year-end assessment of activities and project work (10 + 5) to get the total score out of 20 marks.

Contents

Activities

Projects

Project 1: Efficiency in Packing

To investigate the efficiency of packing of objects of different shapes in a cuboid box. *(Efficiency is the percentage of box space occupied by the objects.)*

Project 2: Geometry in real life

In this project we try to find situations in daily life where geometrical notions can be effectively used. In particular, the student discovers situations in which properties of similar triangles learnt in the classroom are useful.

Project 3: Experiments on Probability

To appreciate that finding probability through experiment is different from finding proability by calculation. Students become sesitive towards the fact that if they increase the number of observaions, probabilty found through experiment approaches the calculated probability.

Project 4: Displacement and rotation of a geometrical figure

To study the distance between different points of a geometrical figure when it is displaced and / or rotated. Enhances familiarity with co-ordinate geometry.

Project 5: Frequency of letters/ words in a language

Analysis of a language text using graphical and pie chart techniques.

Group Activities

Group activity 1

Fourth order Magic Dance

The interplay of mathematics and art can be very appealing. This activity makes an attempt to present a versatile form of the fourth order magic square through a dance.

Group activity 2

Live Lattice

Live lattice is a lattice formed by students placed in square or rectangular formation .

Suggested Projects

Project 1

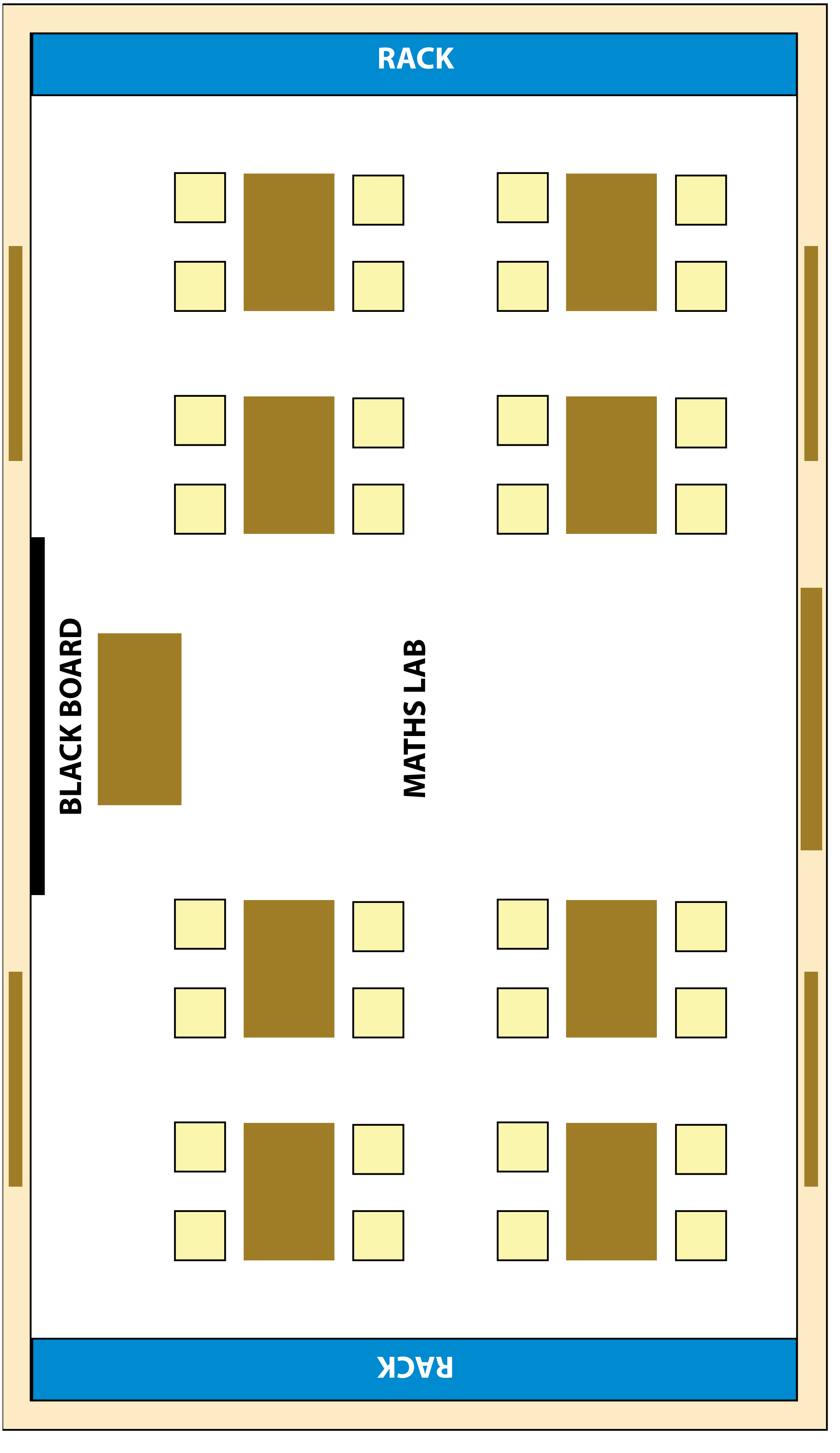
Mathematical designs and patterns using arithmetic progression

Project 2

Early history of Mathematics

Project 3

Analysis of test results and interpretation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MATHS LAB
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Activity 1

System of linear equations

Objective
To obtain the conditions for consistency of a system of linear equations in two variables by graphical method.

Materials required

3 graph papers,
ruler,
pencil.

Pre-requisite knowledge
Plotting of points on a graph paper

Procedure

1. Take the first pair of linear equations in two variables of the form

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

e.g. $2x - 3y = 3$
$3x - 4y = 5$

2. Obtain a table of ordered pairs (x, y), which satisfy the given equation. Find at least three such pairs for each equation.

e.g. For $2x - 3y = 3$

x	0	3	6
y	-1	1	3

For $3x - 4y = 5$

x	- 1	- 5	7
y	- 2	- 5	4

3. Plot the graphs for the two equations on the graph paper as shown in Fig 1(a).

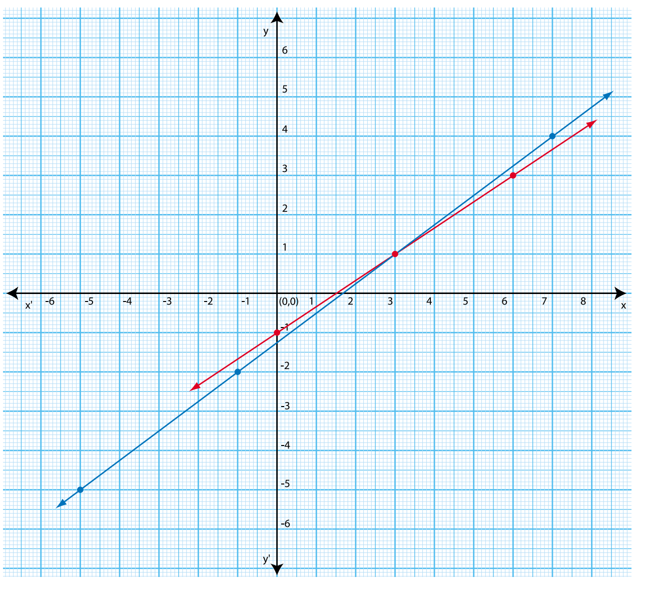


Fig 1(a)

4. Observe if the lines are intersecting, parallel or coincident and note the following:

$$\frac{a_1}{a_2} = \qquad \frac{b_1}{b_2} = \qquad \frac{c_1}{c_2} =$$

5. Take the second pair of linear equations in two variables,
   e.g. $6x + 10y = 4$, $3x + 5y = -11$
6. Repeat the steps from 2 to 4.
7. Take the third pair of linear equations in two variables
   e.g. $x - 2y = 5$, $2x - 4y = 10$
8. Repeat the steps from 2 to 4.
9. Fill in the following observation table

Type of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$
Intersecting			
Parallel			
Coincident			

10. Obtain the condition for two lines to be intersecting, parallel or coincident from the observation table by comparing the values of $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$.

Observations

The student will observe that for intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

for parallel lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ and for coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Learning outcomes

The student will learn that some pairs of linear equations in two variables have a unique solution (intersecting lines); some have infinitely many solutions (coincident lines) and some have no solution (parallel lines).

Remarks

1. The teacher will explain that when a system of linear equations has solution (whether unique or not), the system is said to be consistent; when the system of linear equations has no solution, it is said to be inconsistent.

2. The teacher may consider additional examples in which some of the co-efficients are zero.

Activity 2

Arithmetic Progression - I

Materials required

coloured paper,
pair of scissors,
geometry box,
fevicol,
sketch pens,
one squared paper.

Objective

To verify that the given sequence is an arithmetic progression by paper cutting and pasting method.

Pre-requisite knowledge

1. Definition of an arithmetic progression

Procedure

1. Take a given sequence of numbers say a_1, a_2, a_3 ….
2. Cut a rectangular strip from a coloured paper of width k = 1 cm (say) and length a_1 cm.
3. Repeat this procedure by cutting rectangular strips of the same width k = 1cm and lengths $a_2, a_3, a_4, \ldots$ cm.
4. Take 1 cm squared paper and paste the rectangular strips adjacent to each other in order.

A] Let the sequence be 1, 4, 7, 10, ….

Take strips of lengths 1 cm, 4 cm, 7 cm and 10 cm, all of the same width say 1 cm. Arrange the strips in order as shown in Fig 2(a). Observe that the adjoining strips have a common difference in heights. (In this example it is 3 cm.)

B] Let another sequence be 1, 4, 6, 9, …

Take strips of lengths 1 cm, 4 cm, 6 cm and 9 cm all of the same width say 1 cm. Arrange them in an order as shown in Fig 2(b). Observe that in this case the adjoining strips do not have the same difference in heights.

So, from the figures, it is observed that if the given sequence is an arithmetic progression, a ladder is formed in which the difference between the heights of adjoining steps is constant. If the sequence is not an arithmetic progression, a ladder is formed in which the difference between adjoining steps is not constant.

Learning outcome

Students learn the meaning of an arithmetic progression by relating it to an activity that involves visualization.

Remark

The teacher may point out that in this activity taking width of the strips to be constant is not essential but convenient for visual simplicity of the ladder.

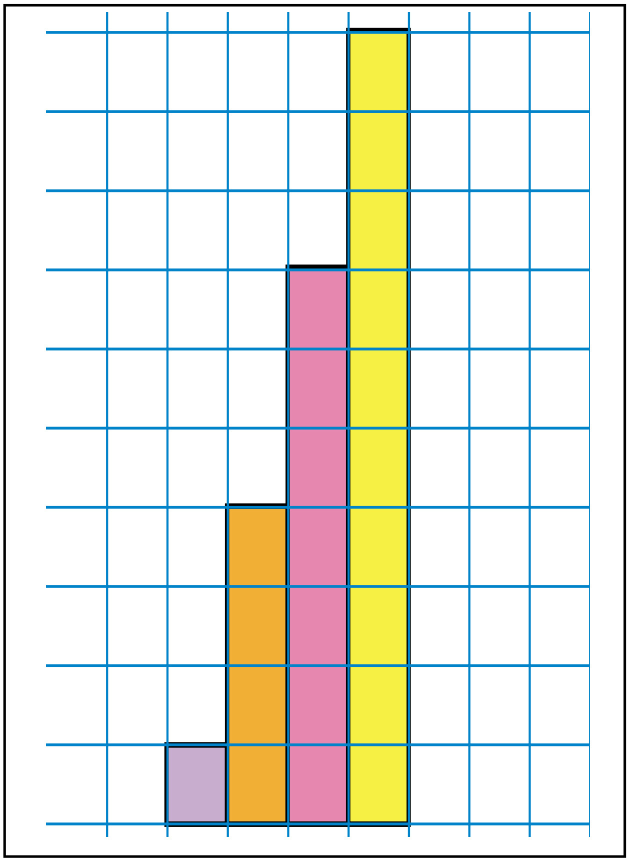


Fig 2(a)

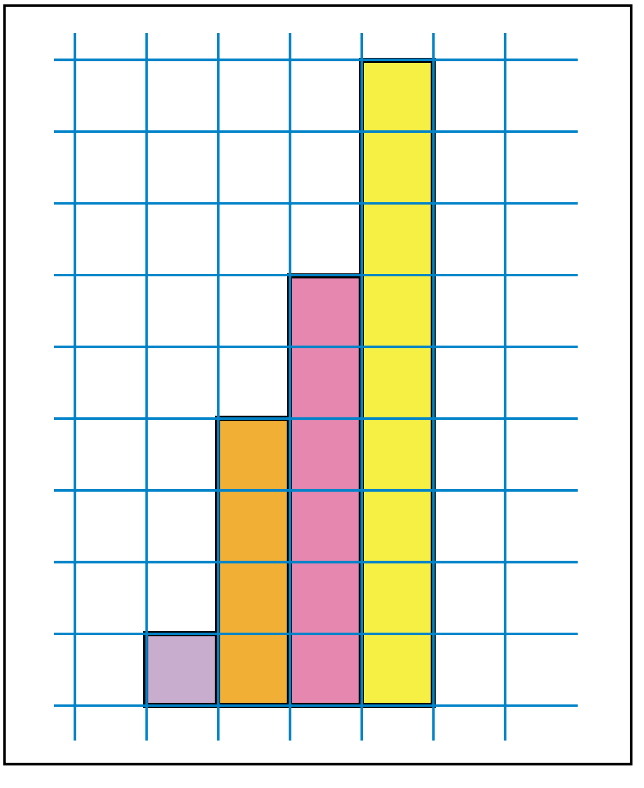


Fig 2(b)

Activity 3

Arithmetic Progression - II

Materials required

chart paper,
sketch pens,
geometry box,
squared paper.

Objective

To verify that the sum of first n natural numbers is $n\,(n + 1) / 2$, i.e. $\Sigma\, n = n\,(n + 1) / 2$, by graphical method.

Pre-requisite knowledge

1. Natural number system
2. Area of squares and rectangles

Procedure

Let us consider the sum of natural numbers say from 1 to 10, i.e. $1 + 2 + 3 + \ldots + 9 + 10$. Here $n = 10$ and $n + 1 = 11$.

1. Take a squared paper of size 10×11 squares and paste it on a chart paper.
2. On the left side vertical line, mark the squares by 1, 2, 3, … 10 and on the horizontal line, mark the squares by 1, 2, 3 …. 11.
3. With the help of sketch pen, shade rectangles of length equal to 1 cm, 2 cm, …, 10cm and of 1 cm width each.

Observations

The shaded area is one half of the whole area of the squared paper taken. To see this, cut the shaded portion and place it on the remaining part of the grid. The student will observe that it completely covers the grid.

Area of the whole squared paper is 10×11 cm ^{2}.
Area of the shaded portion is $(10 \times 11) / 2$ cm^2.
This verifies that, for $n = 10$,

$$\Sigma\, n = n \times (n + 1) / 2$$

The same verification can be done for any other value of n.

Learning outcome

Students develop a geometrical intuition of the formula for the sum of natural numbers starting from one.

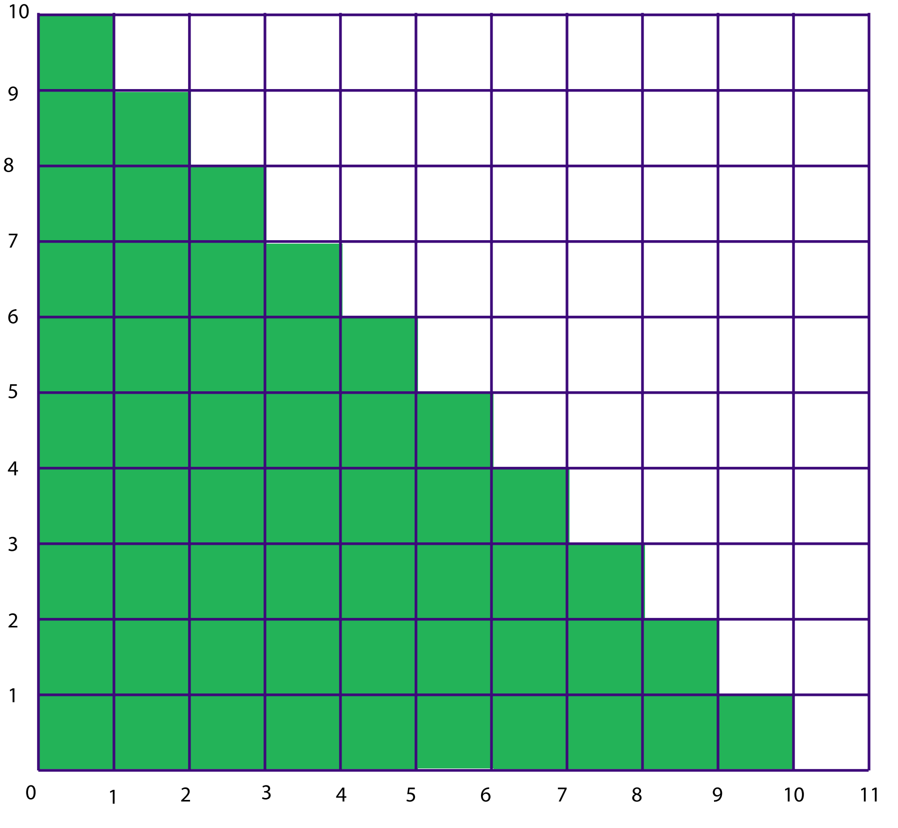


Fig 3

Activity 4

Basic Proportionality Theorem for a Triangle

Materials required

coloured paper,
pair of scissors,
parallel line board,
ruler,
sketch pens.

Objective

To verify the Basic Proportionality Theorem using parallel line board and triangle cut-outs.

> ***Basic Proportionality Theorem***
> *If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.*

Pre-requisite knowledge

Drawing parallel lines on a rectangular sheet of paper.

Procedure

1. Cut three different triangles from a coloured paper. Name them as ΔABC, ΔPQR and ΔDEF.
2. Take the parallel line board (a board on which parallel lines are drawn) as shown in Fig 4 (a). (Note: Students can make the parallel line board, using the techniques given in class IX laboratory manual.)
3. Place ΔABC on the board such that any one side of the triangle is placed on one of the lines of the board as shown in Fig 4 (b). (It would be preferable to place the triangle on the lowermost or uppermost line.)
4. Mark the points P_1, P_2, P_3, P_4 on ΔABC as shown in Fig 4(b).
   Join P_1P_2 and P_3P_4.
   $P_1P_2 \parallel BC$ and $P_3P_4 \parallel BC$
5. Note the following by measuring the lengths of the respective segments using a ruler.

Ratios	Value
$\frac{AP_1}{P_1B}$	
$\frac{AP_2}{P_2C}$	
$\frac{AP_3}{P_3B}$	
$\frac{AP_4}{P_4C}$	

6. Repeat the experiment for ΔDEF and ΔPQR.

Observations

1. Students will observe that

In ΔABC $\quad \dfrac{AP_1}{P_1B} = \dfrac{AP_2}{P_2C}$

$$\frac{AP_3}{P_3B} = \frac{AP_4}{P_4C}$$

2. Students will note similar equalities for ΔDEF and ΔPQR.
3. Students will observe that in all the three triangles the Basic Proportionality Theorem is verified.

Learning outcome

Knowledge of the Basic Proportionality Theorem for a triangle will be reinforced through this activity.

Remark

The teacher will point out to the students to observe that $P_1P_2 \parallel BC$ and $P_3P_4 \parallel BC$ because segments P_1P_2 , P_3P_4 and BC are part of the lines parallel to each other.

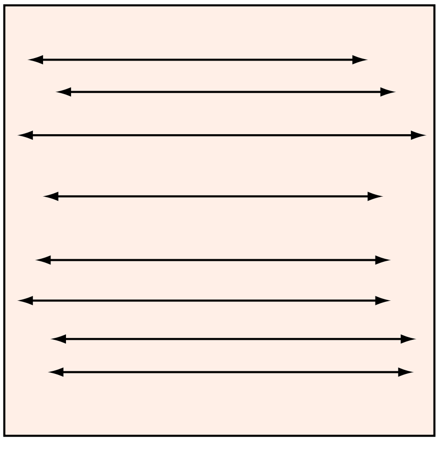


Fig 4(a)

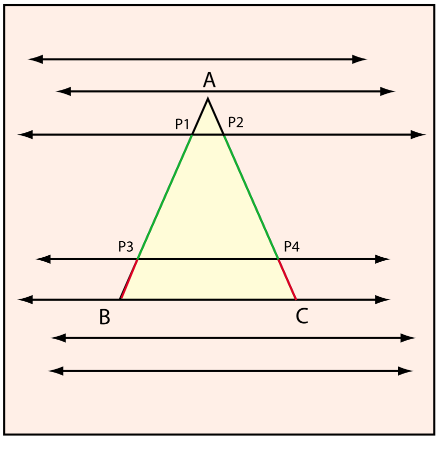


Fig 4(b)

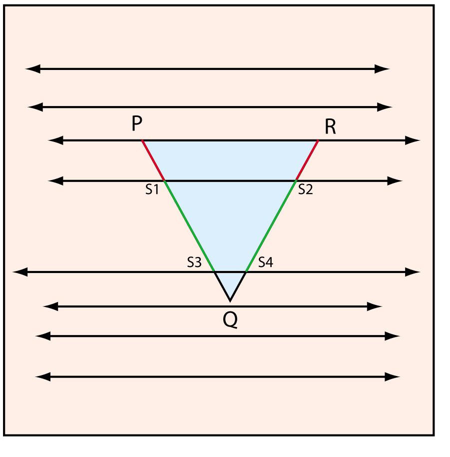


Fig 4(c)

Activity 5

Pythagoras theorem

Materials required

card board,
coloured pencils,
pair of scissors,
fevicol,
geometry box.

Objective

To verify the Pythagoras Theorem by the method of paper folding, cutting and pasting.

> ***Pythagoras Theorem***
> *The area of the square on the hypotenuse of a right angled triangle is equal to the sum of the areas of squares on the other two sides.*

Pre-requisite knowledge

1. Area of a square.
2. Construction of parallel lines and perpendicular bisectors.
3. Construction of a right angled triangle.

Procedure

1. Take a card board of size say 20 cm × 20 cm.
2. Cut any right angled triangle and paste it on the cardboard. Suppose its sides are *a*, *b* and *c*.
3. Cut a square of side *a* cm and place it along the side of length *a* cm of the right angled triangle.
4. Similarly cut squares of sides *b* cm and *c* cm and place them along the respective sides of the right angled triangle.
5. Label the diagram as shown in Fig 5(a).
6. Join BH and AI. These are two diagonals of the square ABIH. The two diagonals intersect each other at the point O.
7. Through O, draw RS || BC.
8. Draw PQ, the perpendicular bisector of RS, passing through O.
9. Now the square ABIH is divided in four quadrilaterals. Colour them as shown in Fig 5(a).
10. From the square ABIH cut the four quadrilaterals. Colour them and name them as shown in Fig 5(b).

Observations

The square ACGF and the four quadrilaterals cut from the square ABIH completely fill the square BCED. Thus the theorem is verified.

Learning Outcome

Students learn one more method of verifying Pythagoras theorem.

Remark

The teacher may point out that the activity only verifies Pythagoras theorem for the given triangle. Verification is different from a general mathematical proof.

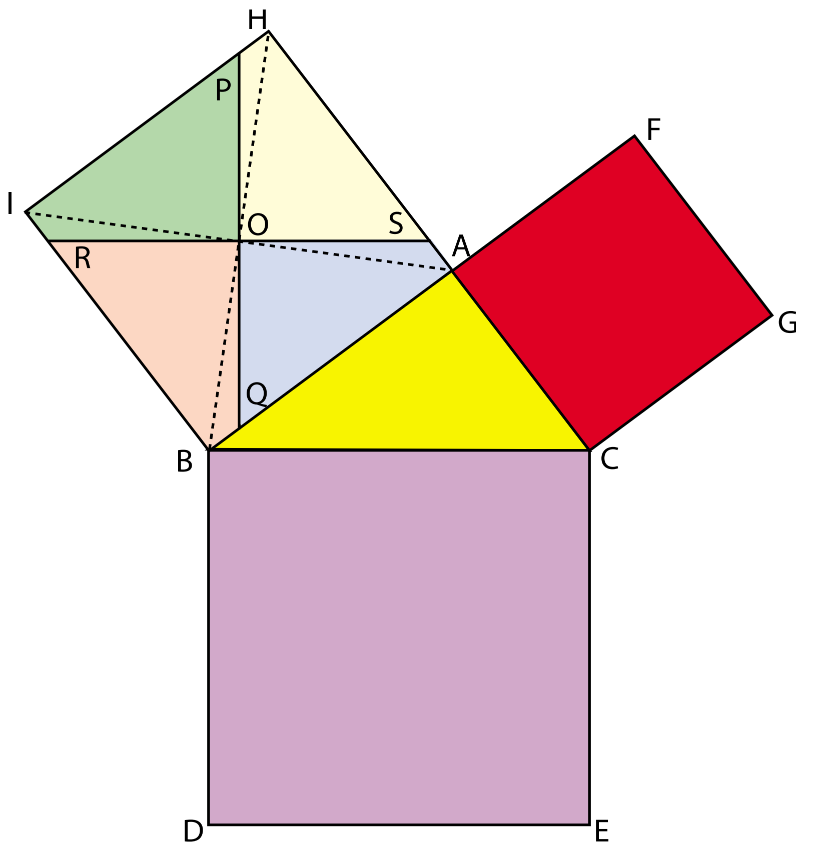


Fig 5(a)

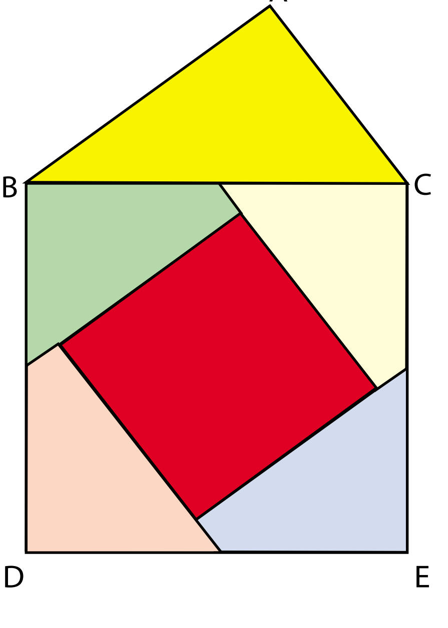


Fig 5(b)

Activity 6

Angle at the centre of a circle by an arc

Materials required

coloured papers,
a pair of scissors,
gum,
compass,
pencil,
ruler,
carbon paper or
tracing paper.

Objective

To verify that the angle subtended by an arc at the centre of a circle is twice the angle subtended by the same arc at any other point on the remaining part of the circle, using the method of paper cutting, pasting and folding.

Pre-requisite knowledge

Meaning of subtended angle by an arc.

Procedure

1. Draw a circle of any radius with centre O on a coloured sheet of paper and cut it.
2. Take a rectangular sheet of paper and paste the cutout on it. [Fig 6(a)]
3. Take two points A and B on the circle to obtain arc AB. [Fig 6(b)]
4. Form a crease joining OA and draw OA. [Fig 6(c)]
5. Form a crease joining OB and draw OB. [Fig 6(d)]
6. Arc AB subtends $\angle$AOB at the centre O of the circle. [Fig 6(e)]
7. Take a point P on the remaining part of the circle.
8. Form a crease joining AP and draw AP. [Fig 6(f)]
9. Form a crease joining BP and draw BP. [Fig 6(g)]
10. Arc AB subtends $\angle$APB at the point P on the remaining part of the circle. [Fig 6(h)]
11. Make two replicas of $\angle$APB using carbon paper or tracing paper. [Fig 6(i)]
12. Place two replicas of $\angle$APB adjacent to each other on $\angle$AOB. What do you observe? [Fig 6(j)]

Observations

1. Students will observe that two replicas of $\angle$APB completely cover $\angle$AOB.
2. $\angle AOB = 2\angle APB$

Learning outcome

Students become more familiar with the theorem (that is proved in classroom) through an activity.

Remarks

1. The teacher may ask the students to perform the activity for the cases where arc AB is a major arc or a semi circular arc.
2. The teacher should point out that the activity only verifies the theorem and is not a proof of the theorem.

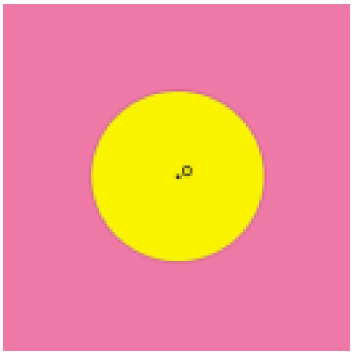


Fig 6(a)

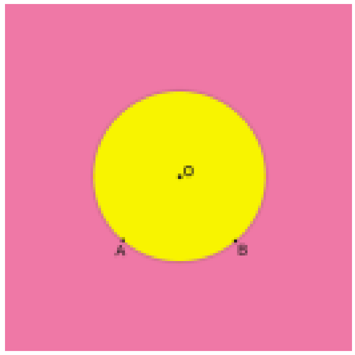


Fig 6(b)

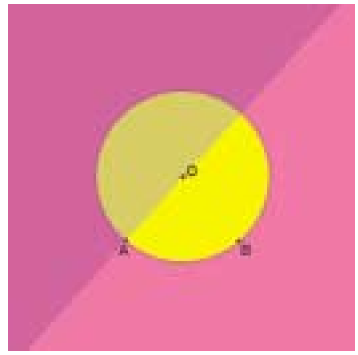


Fig 6(c)

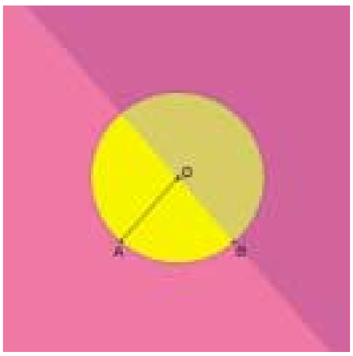


Fig 6(d)

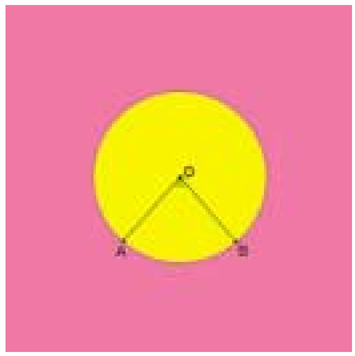


Fig 6(e)

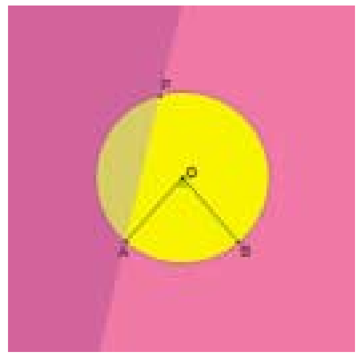


Fig 6(f)

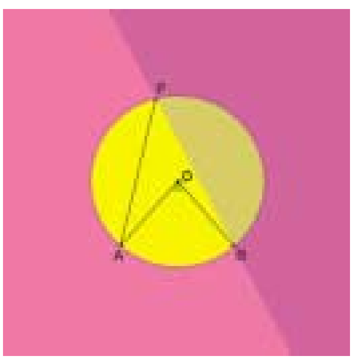


Fig 6(g)

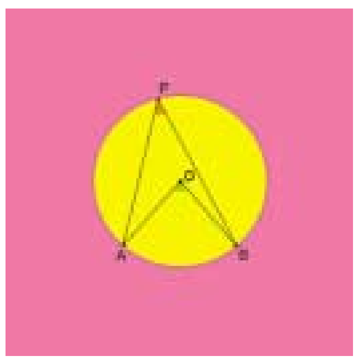


Fig 6(h)

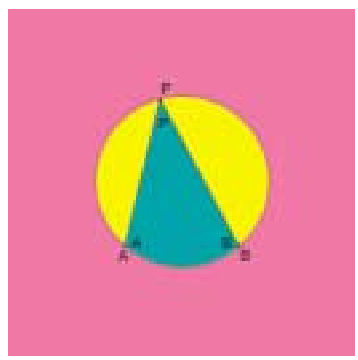


Fig 6(i)

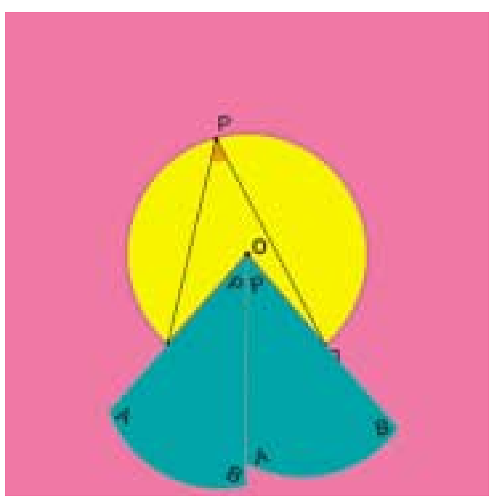


Fig 6(j)

Activity 7

Angles in the same segment

Materials required

coloured papers,
pair of scissors,
gum,
scale,
compass,
pencil,
carbon papers or
tracing papers.

Objective

To verify that the angles in the same segment of a circle are equal, using the method of paper cutting, pasting and folding.

Pre-requisite knowledge

Geometrical meaning of segment of a circle.

Procedure

1. Draw a circle of any radius with centre O and cut it.
2. Paste the cutout on a rectangular sheet of paper. [Fig 7(a)].
3. Fold the circle in any way such that a chord is made. Draw the line segment AB. [Fig 7(b)].
4. Take two points P and Q on the circle in the same segment. [Fig 7(c)].
5. Form a crease joining AP. Draw AP. [Fig 7(d)].
6. Form a crease joining BP. Draw BP. [Fig 7(e)].
7. $\angle$APB is formed in the major segment. [Fig 7(f)]
8. Form a crease joining AQ. Draw AQ. [Fig 7(g)]
9. Form a crease joining BQ. Draw BQ. [Fig 7(h)]
10. $\angle$APB and $\angle$AQB are formed in the major segment. [Fig 7(i)]
11. Make replicas of $\angle$APB and $\angle$AQB using carbon paper or tracing paper. [Fig 7(j)]
12. Place the cutout of $\angle$APB on the cutout of $\angle$AQB. What do you observe?

Observations

Students will observe that

1. $\angle$APB and $\angle$AQB are angles in the same segment.
2. $\angle$AQB covers $\angle$APB exactly. Therefore $\angle APB = \angle AQB$

Learning outcome

Students become more familiar with this theorem (proved in the classroom) through an activity.

Remarks

1. The teacher may ask the student to perform the activity by taking different points P and Q including those very close to points A and B.
2. The teacher may ask the student to perform the activity using point P in one segment and Q in the other segment and note that the angles in the segments are not equal, except in the case when the chord AB is a diameter of the circle. They will find that the two angles are supplementary angles.

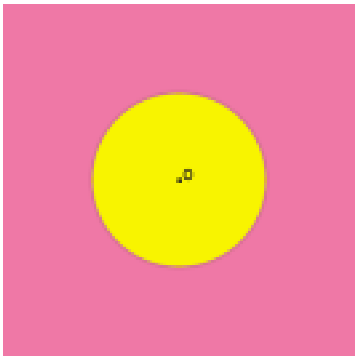


Fig 7(a)

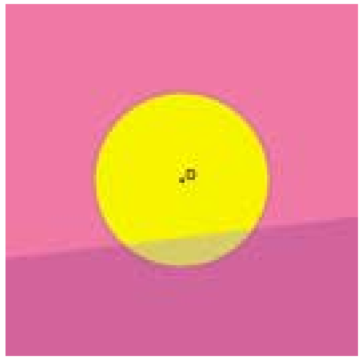
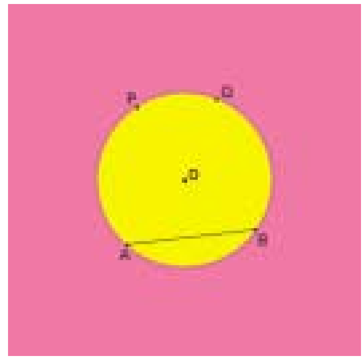


Fig 7(b)



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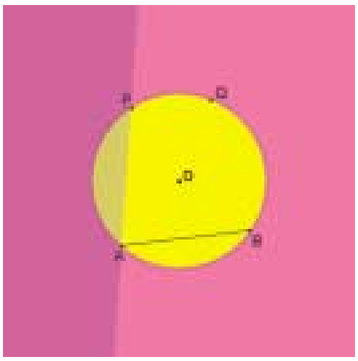

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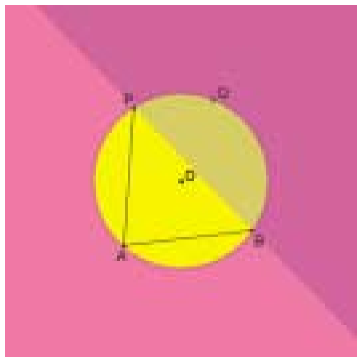


Fig 7(e)

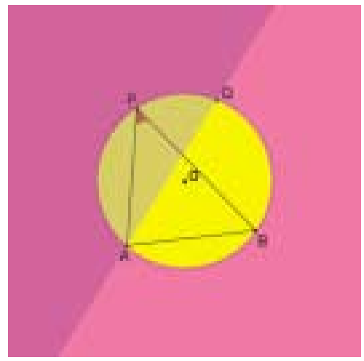


Fig 7(f)

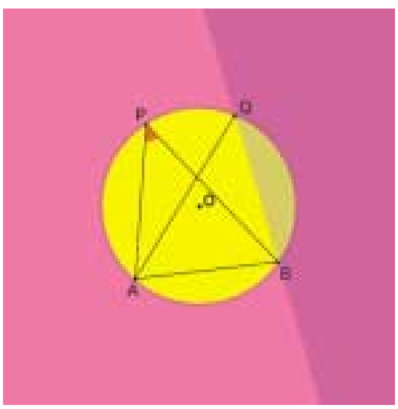


Fig 7(g)

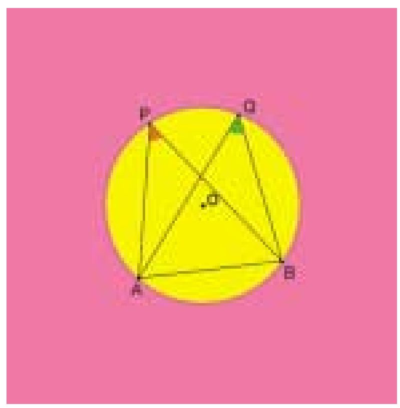


Fig 7(h)

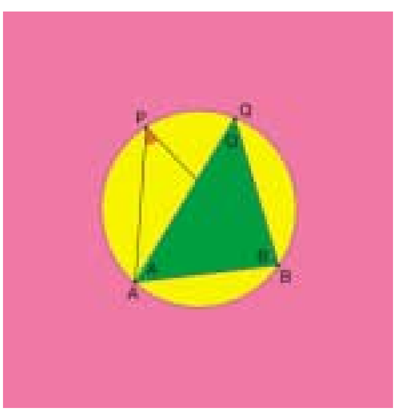


Fig 7(i)

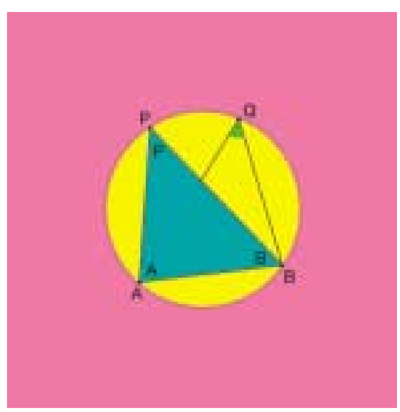


Fig 7(j)

Activity 8

Angle in a semicircle, major segment and minor segment

Materials required

coloured papers,
a pair of scissors,
gum,
scale,
compass,
pencil,
carbon paper or
tracing paper,
cutout of right angled
triangle.

Objective

To verify, using the mrthod of paper cutting, pasting and folding that

(a) the angle in a semicircle is a right angle,
(b) the angle in a major segment is acute,
(c) the angle in a minor segment is obtuse.

Pre-requisite knowledge

Concept of right angle, acute angle, obtuse angle, linear pair axiom.

Procedure for (a)

1. Draw a circle of any radius with centre O on a coloured sheet of a paper and cut it. [Fig 8a(a)]
2. Form a crease passing through the centre O of the circle. Diameter AB is obtained. Draw AB. [Fig 8a(b)]
3. Take a point P on the semicircle.
4. Form a crease joining AP. Draw AP. [Fig 8a(c)]
5. Form a crease joining BP. Draw BP. [Fig 8a(d)]
6. Make two replicas of $\angle$APB; call them $\angle A_1P_1B_1$ and $\angle A_2P_2B_2$. [Fig 8a(e)]
7. Place the two replicas adjacent to each other such that A_1P_1 and P_2B_2 coincide with each other as shown in Fig 8a(f).

Observations

Students will observe that the two line segments P_1A_1 and P_2B_2 lie on a straight line.

Therefore, $\angle A_2P_2B_2 + \angle B_1P_1A_1 = 180^0$

But $\angle A_2P_2B_2$ and $\angle B_1P_1A_1$ are replicas of $\angle$APB.

i.e. $2\angle APB = 180^0$

i.e. $\angle APB = 90^0$

Procedure for (b)

1. Draw a circle of any radius with centre O on a coloured sheet of paper and cut it.
2. Paste the cutout on a rectangular sheet of paper. [Fig 8b(a)]
3. Fold the circle in such a way that a chord AB is obtained. Draw AB. [Fig 8b(b)]
4. Take a point P in the major segment.
5. Form a crease joining AP. Draw AP. [Fig 8b(c)]
6. Form a crease joining BP. Draw BP. [Fig 8b(d)]
7. Make a replica of $\angle$APB. [Fig 8b(e)].
8. Place the replica of $\angle$APB on a right angled Δ DEF such that BP falls on DE. [Fig 8b(f)]. What do you observe?

Observations

Students will observe that

1. $\angle$BPA does not cover $\angle$DEF completely. [Fig 8b(f)]
2. $\angle$BPA is smaller that the $\angle$DEF.
3. $\angle$DEF is 90^0.
4. Therefore, $\angle$BPA is acute.
5. $\angle$BPA is an angle in the major segment.

Procedure for (c)

1. Draw a circle of any radius with centre O on a coloured sheet of paper and cut it.
2. Paste the cutout on a rectangular sheet of paper. [Fig 8c(a)]
3. Fold the circle in such a way that a chord AB is obtained. Draw AB. [Fig 8c(b)]
4. Take a point P in the minor segment.
5. Form a crease joining AP. Draw AP. [Fig 8c(c)]
6. Form a crease joining BP. Draw BP. [Fig 8c(d)]
7. Make a replica of $\angle$APB. [Fig 8c(e)]
8. Place the right angled Δ DEF on the replica of $\angle$APB such that DE falls on BP. [Fig 8c(f)] What do you observe?

Observations

Students will observe that

1. $\angle$ DEF does not cover $\angle$ BPA completely. [Fig 8c(f)]
2. $\angle$ DEF is smaller that the $\angle$ BPA.
3. $\angle$DEF is 90^0.
4. Therefore, $\angle$BPA is obtuse.
5. $\angle$BPA is an angle in the minor segment.

Learning outcome

Students develop familiarity with the fact that the angle in a semicircle is right angle, the angle in a major segment is acute angle and the angle in a minor segment is obtuse angle.

Remark

The teacher may point out that for a given chord AB, the obtuse angle in the minor segment and the acute angle in the major segment are supplementary angles. The students may be asked to verify this by taking appropriate cutouts of the angles.

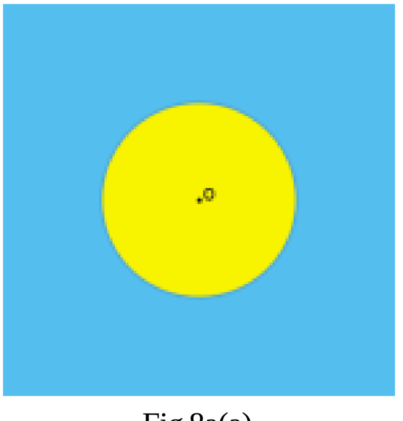


Fig 8a(a)

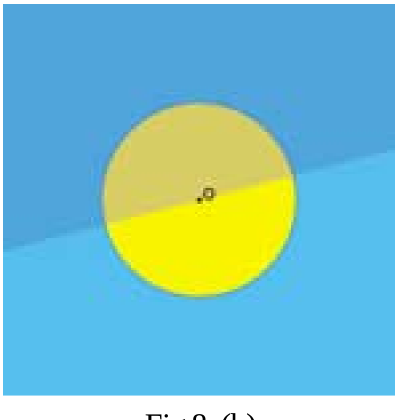


Fig 8a(b)

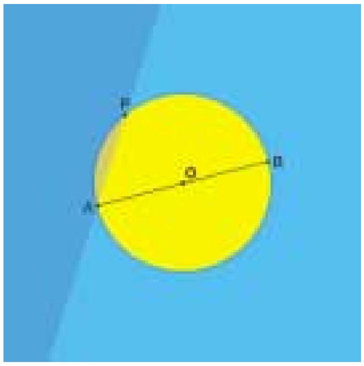


Fig 8a(c)

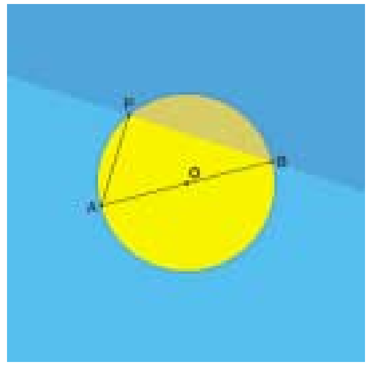


Fig 8a(d)

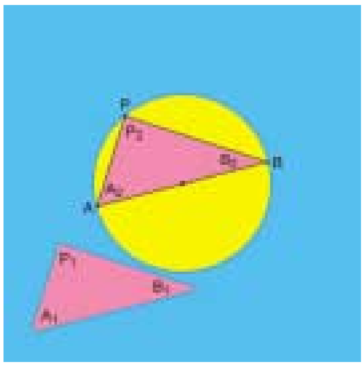


Fig 8a(e)

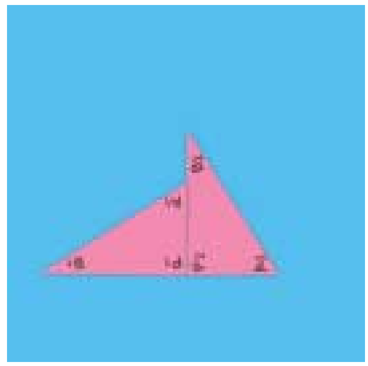


Fig 8a(f)

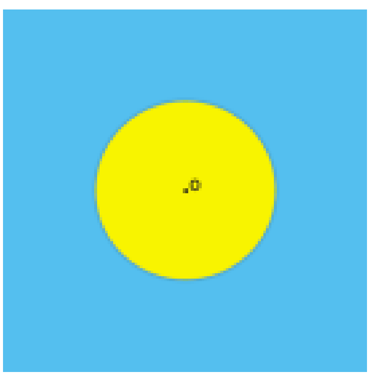


Fig 8b(a)

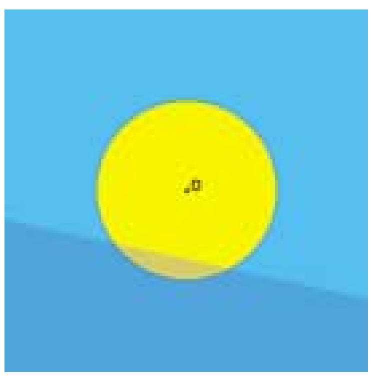


Fig 8b(b)

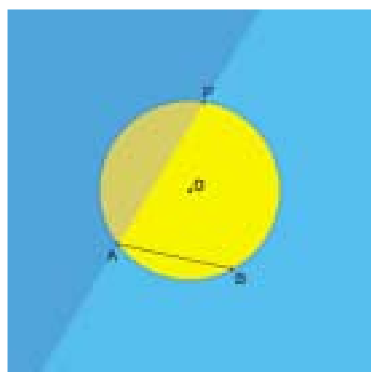


Fig 8b(c)

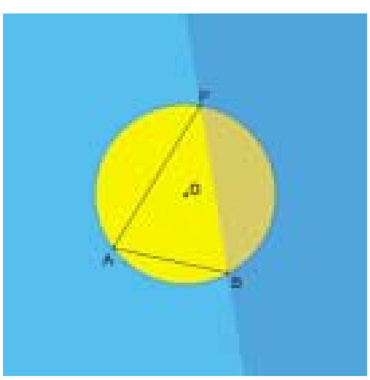


Fig 8b(d)

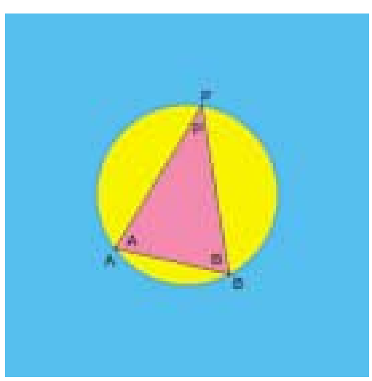


Fig 8b(e)

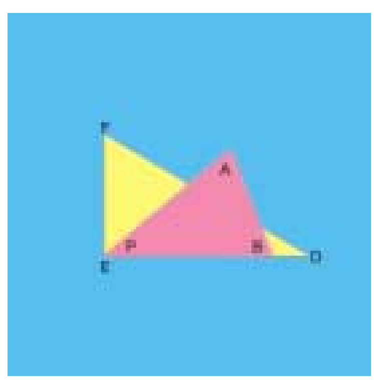


Fig 8b(f)

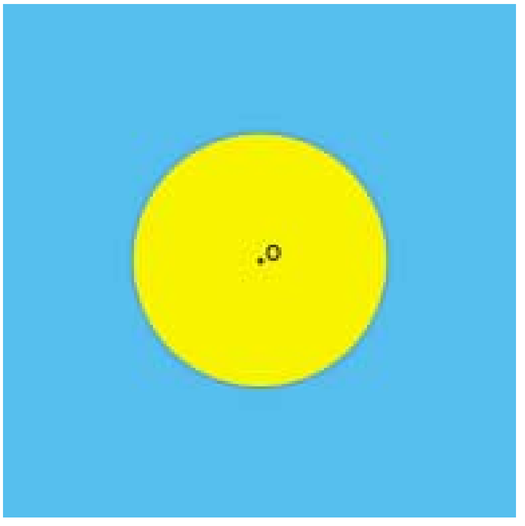


Fig 8c(a)

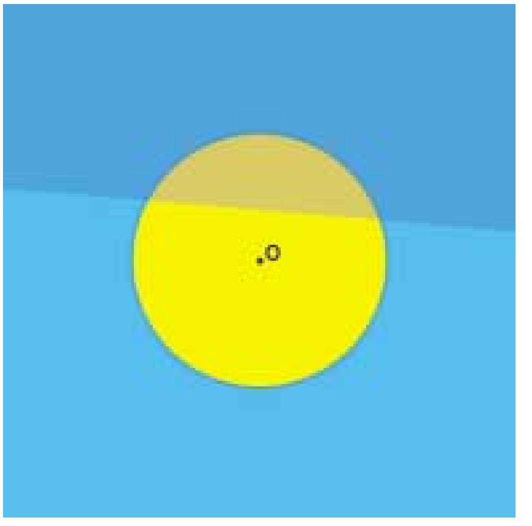


Fig 8c(b)

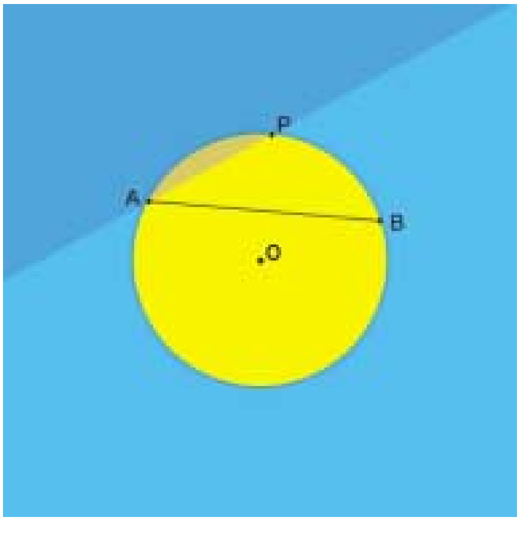


Fig 8c(c)

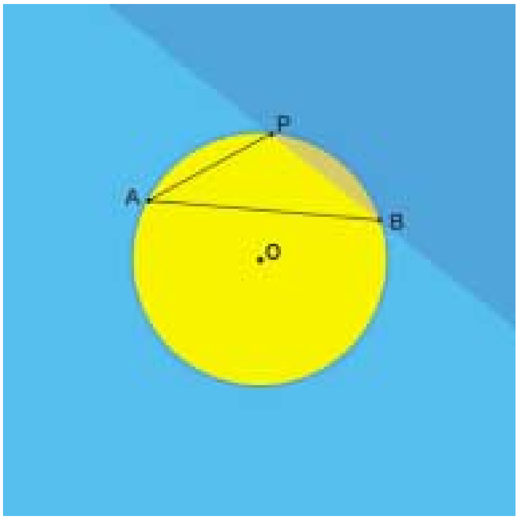


Fig 8c(d)

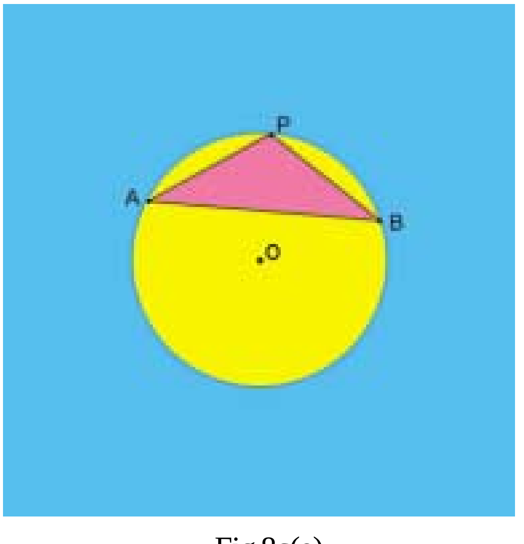


Fig 8c(e)

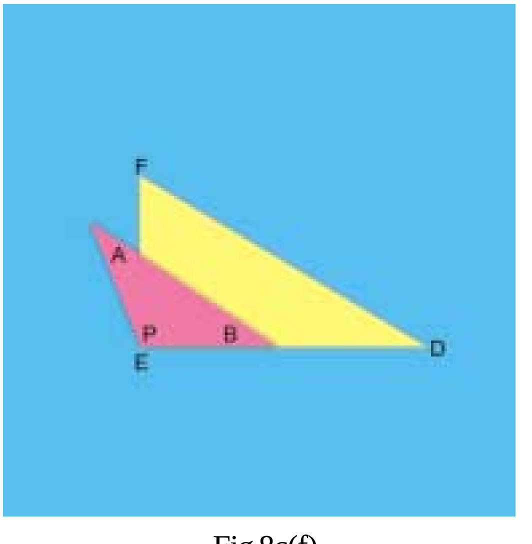


Fig 8c(f)

Activity 9

Cyclic Quadrilateral Theorem

Materials required

coloured papers,
pair of scissors,
ruler,
sketch pen,
carbon paper or
tracing paper.

Objective

To verify, using the method of paper cutting, pasting and folding that

1. the sum of either pair of opposite angles of a cyclic quadrilateral is 180^0.
2. in a cyclic quadrilateral the exterior angle is equal to the interior opposite angle.

Pre-requisite knowledge

1. Meaning of cyclic quadrilateral, interior opposite angle
2. Linear pair axiom

Procedure

1. Draw a circle of any radius on a coloured paper and cut it.
2. Paste the cutout on a rectangular sheet of paper.[Fig 9(a)].
3. By paper folding get chords AB, BC, CD and DA.
4. Draw AB, BC, CD and DA. Cyclic quadrilateral ABCD is obtained [Fig 9(b)].
5. Make a replica of cyclic quadrilateral ABCD using carbon paper / tracing paper. [Fig 9(c)]
6. Cut the quadrilateral cutout into four parts such that each part contains one angle i.e. $\angle$A, $\angle$B, $\angle$C and $\angle$D. [Fig 9(d)]
7. Place $\angle$A and $\angle$C adjacent to each other. What do you observe? [Fig 9(e)]
8. Produce AB to form a ray AE. Exterior angle $\angle$CBE is formed. [Fig 9(f)]
9. Place the replica of D on $\angle$CBE. [Fig 9(g)] What do you observe?

Observations

Students will observe that

1. When $\angle$A and $\angle$C are placed adjacent to each other they form a linear pair. This shows $\angle A + \angle C = 180^0$
2. $\angle$D completely covers $\angle$CBE. This shows that exterior angle of a cyclic quadrilateral ABCD is equal to the interior opposite angle.

Learning outcome

Students develop geometrical intuition of the result that

1. opposite angles of a cyclic quadrilateral are supplementary.
2. exterior angle to a cyclic quadrilateral is equal to the interior opposite angle.

Remarks

1. The teacher may ask the students to perform the activity for the other pair of angles (i.e. $\angle$B and $\angle$D) and for the other exterior angles also.
2. The teacher should point out that this theorem is true only for a cyclic quadrilateral. Students may be asked to perform a similar activity for a non-cyclic quadrilateral.

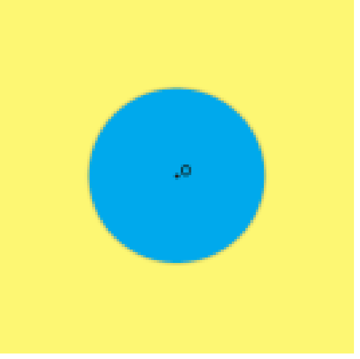


Fig 9(a)

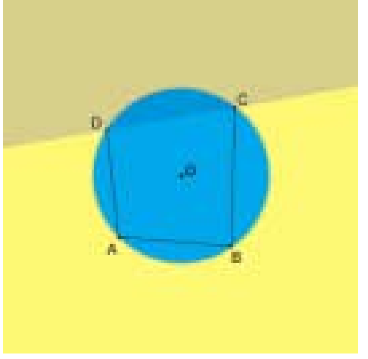


Fig 9(b)

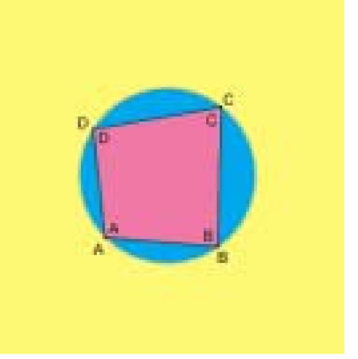


Fig 9(c)

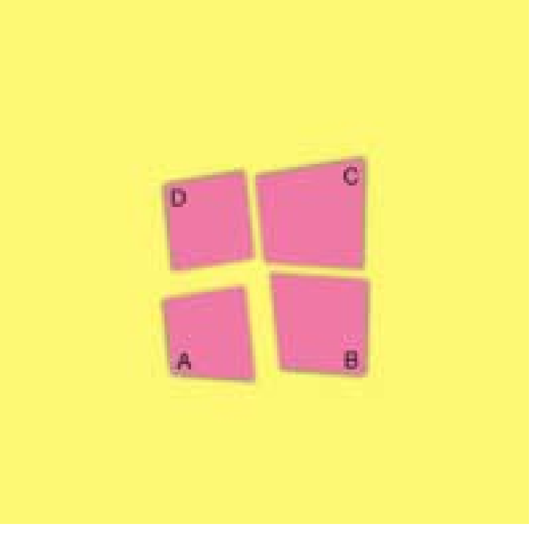


Fig 9(d)

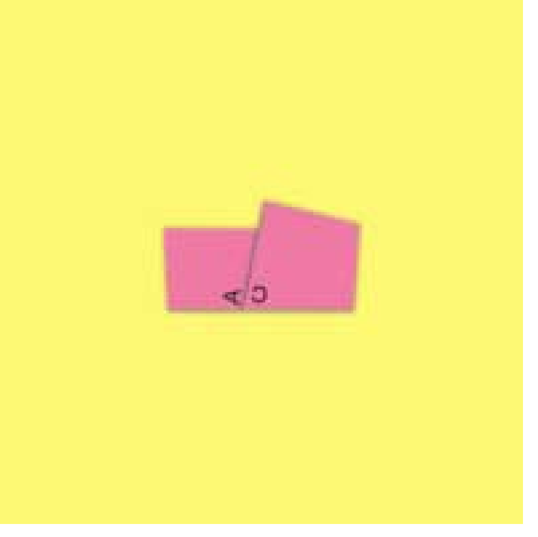


Fig 9(e)

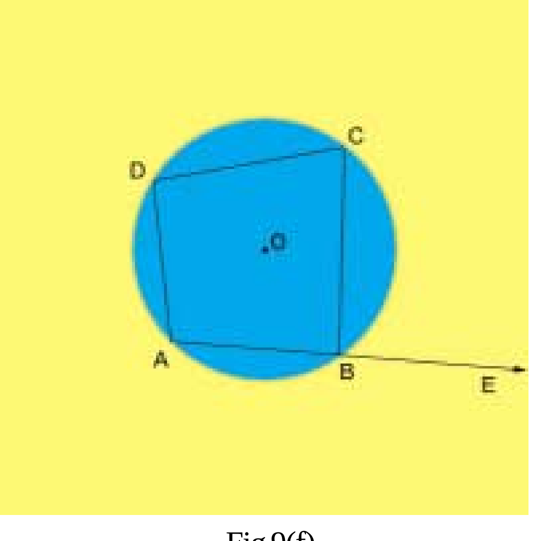


Fig 9(f)

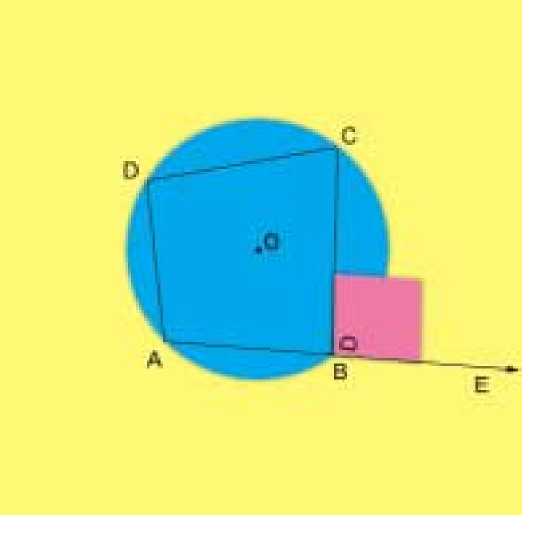


Fig 9(g)

Activity 10

Tangents drawn from an external point

Materials required

coloured papers,
pair of scissors,
ruler,
sketch pens,
compass,
pencil.

Objective

To verify using the method of paper cutting, pasting and folding that the lengths of tangents drawn from an external point are equal.

Pre-requisite knowledge

Meaning of tangent to a circle.

Procedure

1. Draw a circle of any radius on a coloured paper and cut it. Let O be its centre.
2. Paste the cutout on a rectangular sheet of paper.[Fig 10(a)]
3. Take any point P outside the circle.
4. From P fold the paper in such a way that it just touches the circle to get a tangent PA (A is the point of contact). [Fig 10(b)]. Join PA.
5. Repeat step 4 to get another tangent PB to the circle (B is the point of contact). [Fig 10(c)]. Join PB.
6. Join the centre of the circle O to P, A and B. [Fig 10(d & e)]
7. Fold the paper along OP. [Fig 10(f)] What do you observe?

Observations

Students will observe that

1. Δ OPA and Δ OPB completely cover each other.
2. Length of tangent PA = Length of tangent PB.

Learning outcome

Students learn how to get tangents from an external point to a circle using paper folding and verify the theorem.

Remark

The teacher may ask the students to perform the activity by taking point P (external point) at different locations.

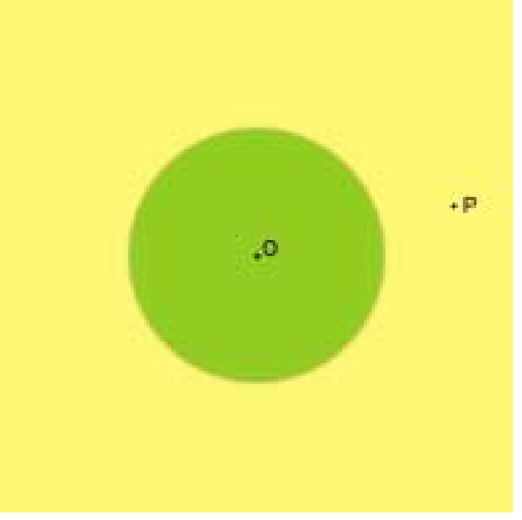


Fig 10(a)

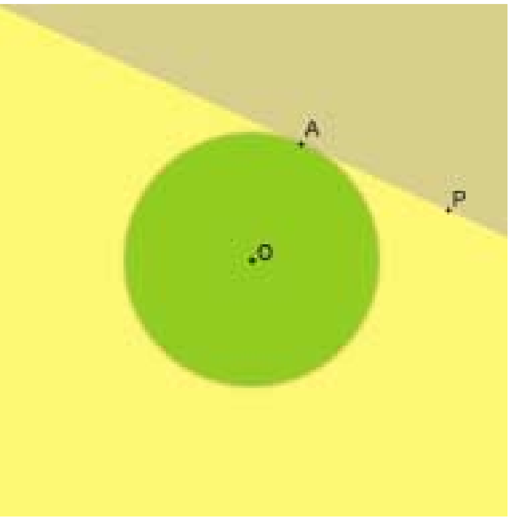


Fig 10(b)

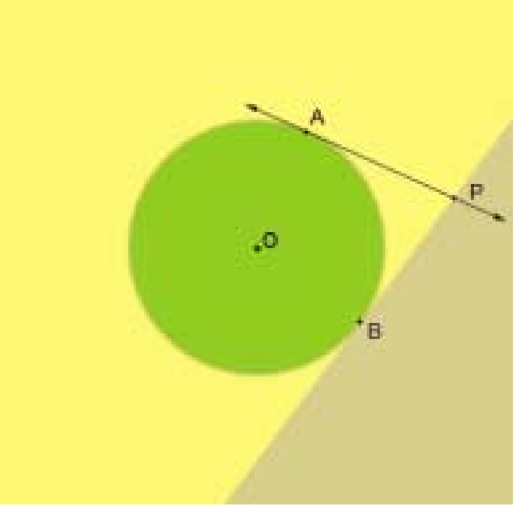


Fig 10(c)

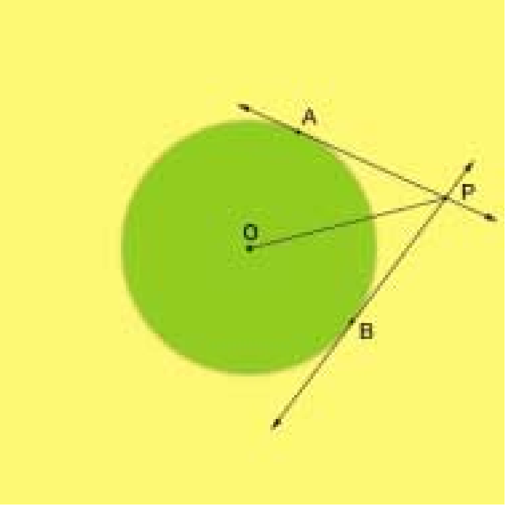


Fig 10(d)

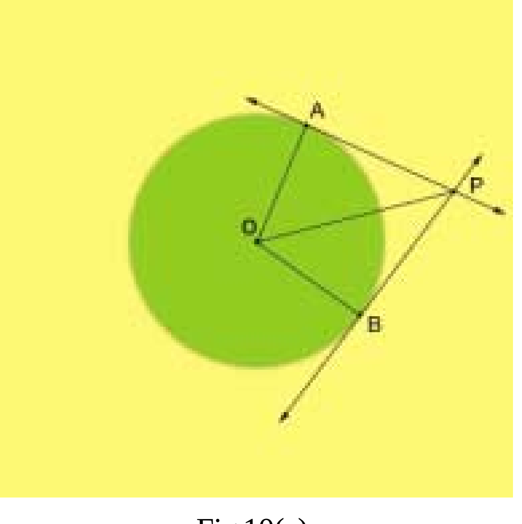


Fig 10(e)

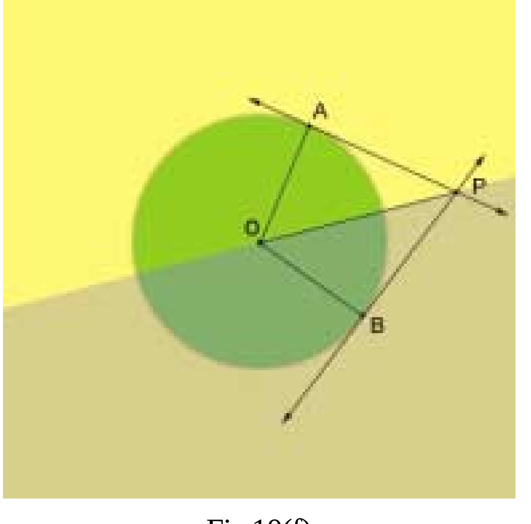


Fig 10(f)

Activity 11

Alternate Segment Theorem

Materials required

coloured paper,
pair of scissors,
ruler,
sketch pen,
fevistik,
carbon paper or
tracing paper.

Objective

To verify the Alternate Segment Theorem by the method of paper cutting, pasting and folding.

> ***Alternate Segment Theorem***
> *If a chord is drawn through the point of contact of a tangent to a circle, then the angles which this chord makes with the given tangent are equal respectively to the angles formed in the corresponding alternate segments.*

Pre-requisite knowledge

Geometrical terms related to a circle.

Procedure

1. Draw a circle of any radius on a coloured paper and cut it.
2. Paste the cut out on a rectangular sheet. [Fig 11(a)]
3. Fold the sheet of paper in such a way that it just touches the circle.[Fig 11(b)]
4. Unfold the paper and draw tangent PQ. Let A be the point of contact as shown in Fig 11(c).
5. Fold the paper starting from A such that chord AB is obtained. Draw AB. [Fig 11(d)].
6. Observe the angles formed by the chord AB and the tangent PQ: $\angle$BAP and $\angle$BAQ.
7. Take a point C on the major arc.
   Form a crease joining AC. Draw AC.[Fig 11(e)]
   Form a crease joining BC. Draw BC. [Fig 11(f)]
8. Take a point D on the minor arc.
   Form a crease joining AD. Draw AD.[Fig 11(g)]
   Form a crease joining BD. Draw BD. [Fig 11(h)]
9. Make a replica of $\angle$ACB, using a carbon / tracing paper. [Fig 11(i)]
   Place it on $\angle$BAQ. [Fig 11(j)]
   What do you observe?
10. Make a replica of $\angle$BDA. [Fig 11(k)]
    Place it on $\angle$BAP. [Fig 11(l)]
    What do you observe?

Observations

1. Students will observe that the chord AB is making two angles $\angle$BAQ and $\angle$BAP with the tangent PQ.
2. They will also observe that replica of $\angle$ACB completely covers $\angle$BAQ and replica of $\angle$ADB completely covers $\angle$BAP. They will then verify the theorem.

Learning outcome

Students will enhance their familiarity with the Alternate Segment Theorem through an activity.

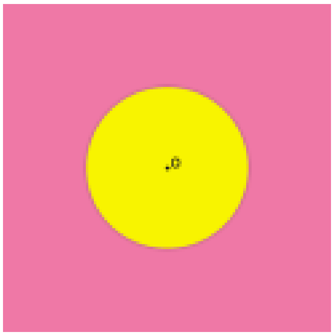


Fig 11(a)

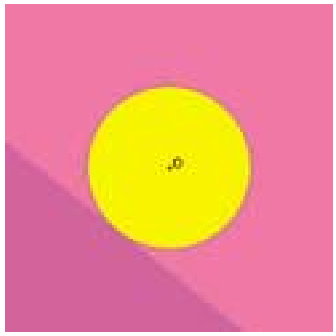


Fig 11(b)

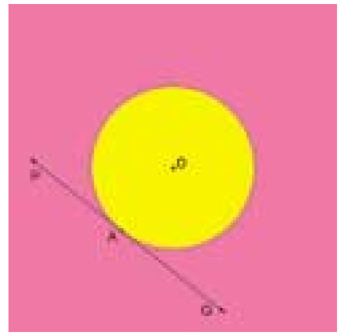
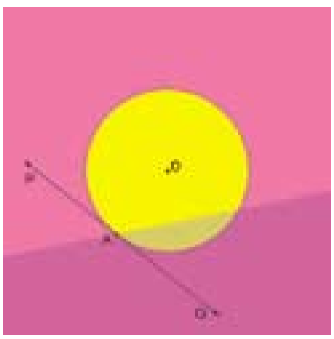


Fig 11(c)



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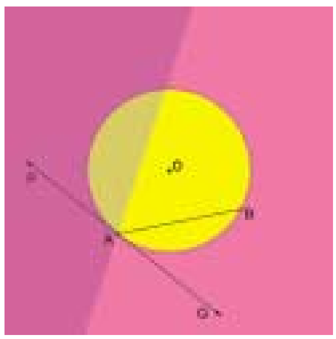

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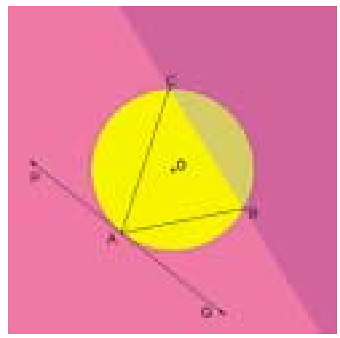


Fig 11(f)

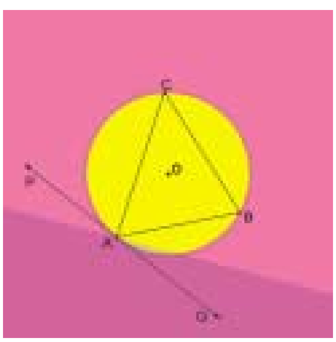


Fig 11(g)

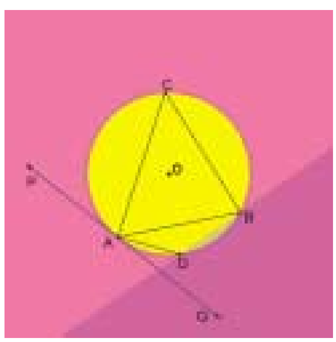


Fig 11(h)

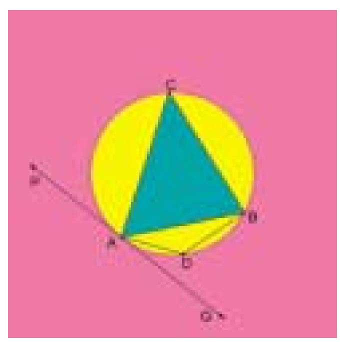
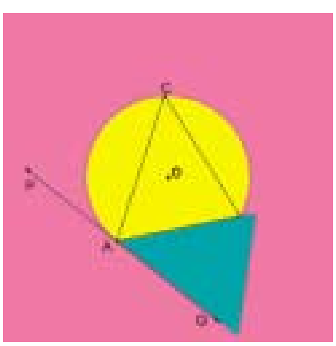


Fig 11(i)



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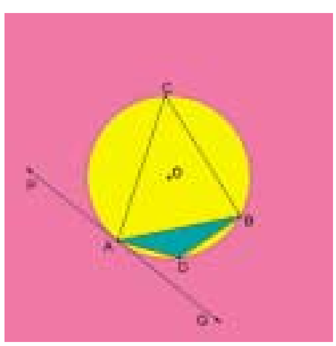


Fig 11(k)

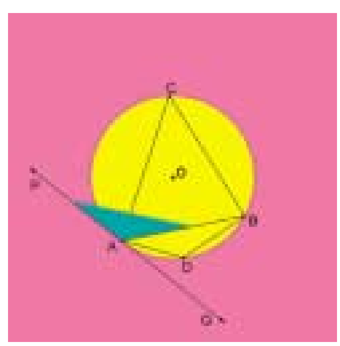


Fig 11(l)

Activity 12

Right circular cylinder

> **Materials required**
>
> coloured papers,
> a pair of scissors,
> gum.

Objective

To make a right circular cylinder of given height and circumference of base.

Pre-requisite knowledge

1. Drawing and cutting a rectangle of given dimensions.
2. Formula for the circumference of a circle.

Procedure

1. Cut a rectangular sheet of paper of length l = given circumference of base of cylinder, breadth b = given height. [Fig 12(a)].
2. Gently curve the paper so that the two (shorter) sides come together.
3. Join the edges together by cello tape.[Fig 12(b)].

Observations

1. The rectangle transforms into a cylinder.
2. The height of the cylinder is b.
3. The circumference of the base circle is l.

Learning outcomes

1. Students learn to make a cylinder of given height and base circumference.
2. Students appreciate how folding of geometrical figures transforms their shape.

Remark

The teacher may suggest to students that b and l may be interchanged to form a different cylinder.

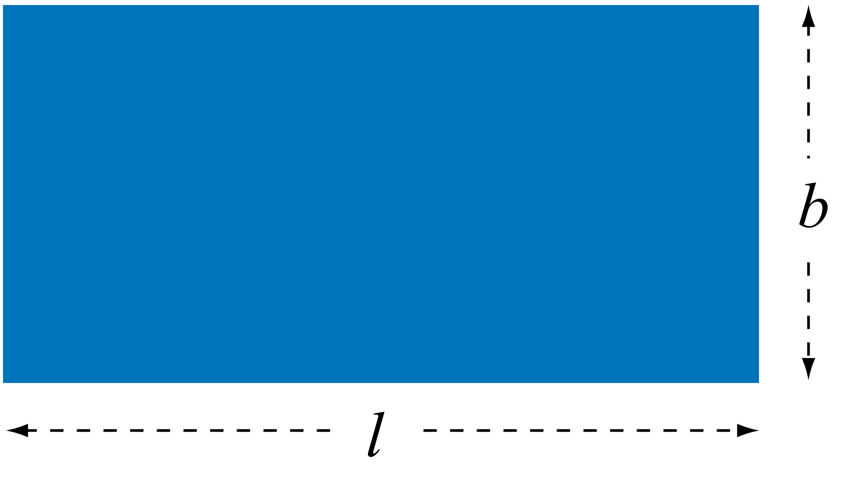


Fig 12(a)

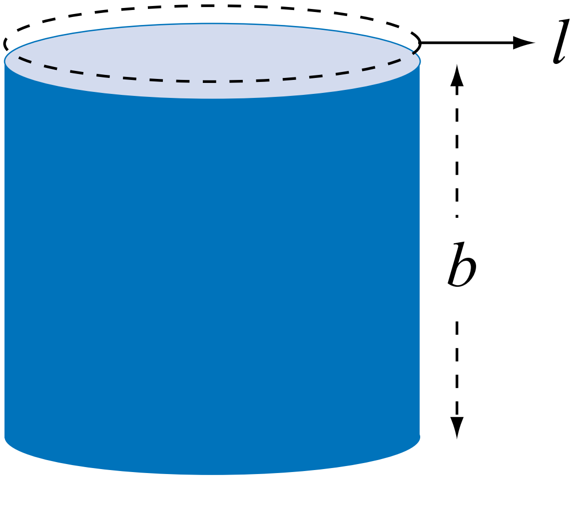


Fig 12(b)

Activity 13

Surface area of a cylinder

Materials required

cylinder of known dimension made of chartpaper,
pair of scissors,
gum,
ruler.

Objective

1. To determine the area of a given cylinder.
2. To obtain the formula for the lateral surface area of a right circular cylinder in terms of the radius (r) of its base and height (h).

Pre-requisite knowledge

1. A rectangle can be rolled to form a cylinder.
2. Area of a rectangle.
3. Area of a circle.

Procedure

1. Remove the top and bottom circles of the cylinder. [Fig 13(b)]
2. Make a vertical cut in the curved surface and lay the cylinder flat. [Fig 13(b)]
3. Measure the length and breadth of the rectangle so formed.

Observations

1. The base and top of the cylinder are congruent circular regions.
2. The curved surface area of the cylinder opens to form a rectangular region.
3. The breadth of the rectangle is the height of the cylinder.
4. The length of the rectangle is the circumference of the base of the cylinder.
5. Curved surface area of cylinder (c) = area of rectangle

   $c = l \times b$

   Since $l = 2\pi r$

   $b = h$

   $c = l \times b = 2\pi rh$
6. Total surface area of cylinder = curved surface area (c) + 2 (area of base circle)

   $= 2\pi rh + 2\pi r^2$

   $= 2\pi r\,(h + r)$

Learning outcome

Students appreciate the derivation of the formula for the curved surface area of a right circular cylinder.

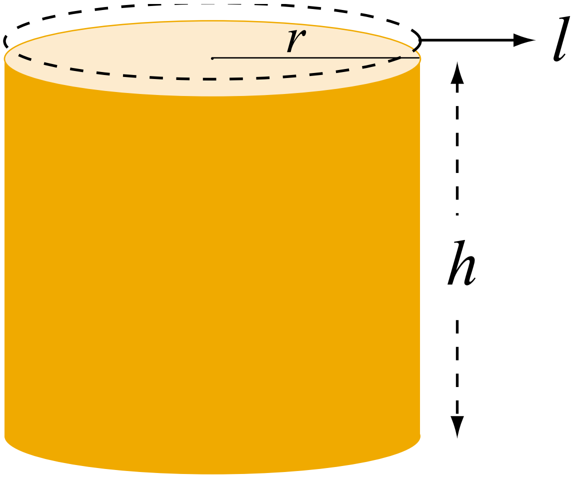


Fig 13(a)

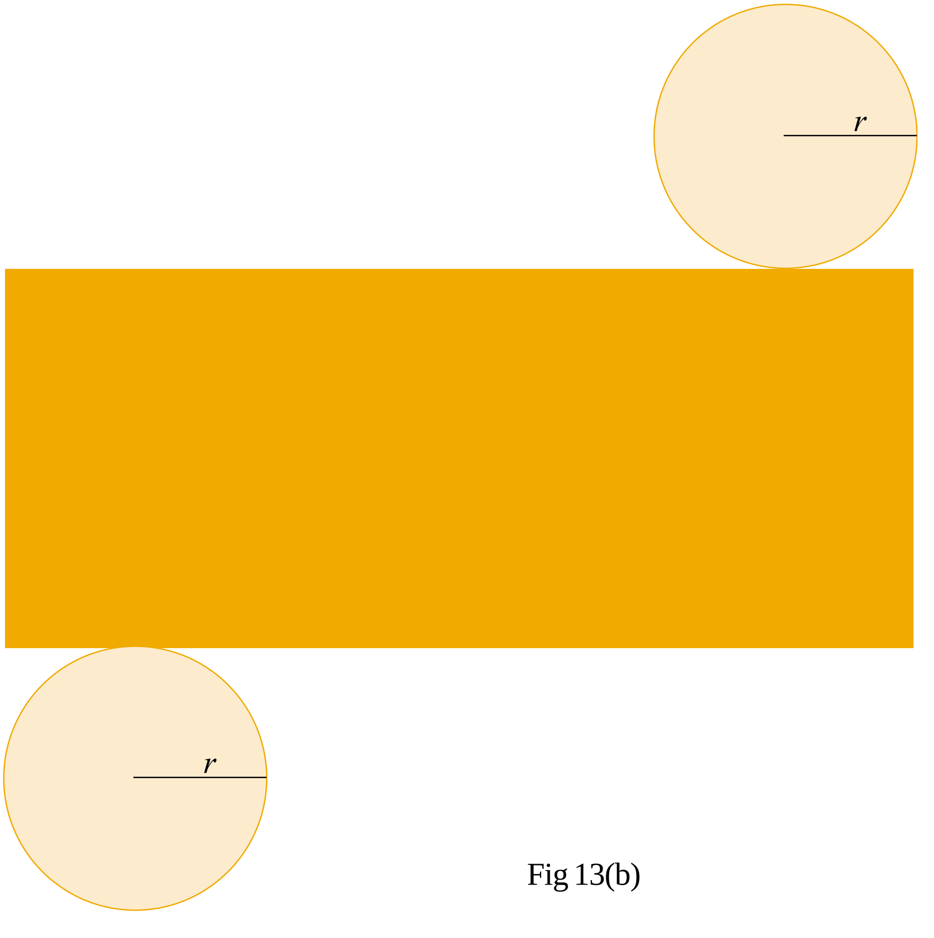


Fig 13(b)

Activity 14

Volume of a right circular cylinder

Materials required

Thermacol,
plastic clay.

Objective

To give a suggestive demonstration of the formula for the volume of a right circular cylinder in terms of its height and radius of the base circle.

Pre-requisite knowledge

1. Formula for volume of a cuboid
2. Formula for circumference of a circle.

Procedure

1. Make a cylinder of any dimensions using plastic clay. Let its height be h and radius of base circle r.
2. Cut the cylinder into 8 sectorial sections as shown in the figure. [Fig 14(a)].
3. Place the segments alternately as shown in the figure. [Fig 14(b)].

Observations

The students observe that

1. The segments approximately form a solid cuboid of height 'h', breadth 'r' and length 'πr'.
2. The volume of the cuboid is $lbh = \pi r \times r \times h = \pi r^2 h$

Learning outcome

Students learn that the volume of a cylinder is $\pi r^2 h$ where r is the radius of the base and h the height of the cylinder.

Remarks

1. The teacher may help the student observe that the length of the cuboid is half the circumference of the base of the cylinder.
2. The teacher should point out that this activity does not give an exact proof of the formula and that the approximation improves by increasing the number of sectorial sections.

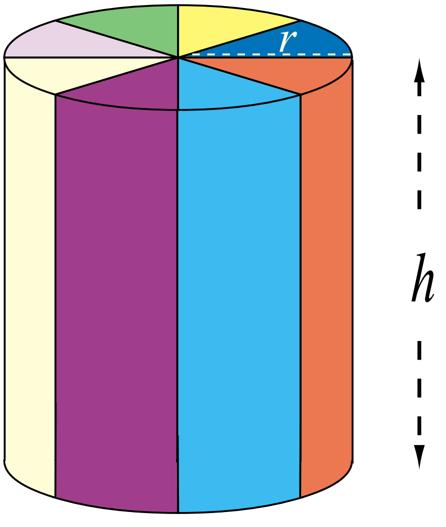


Fig 14(a)

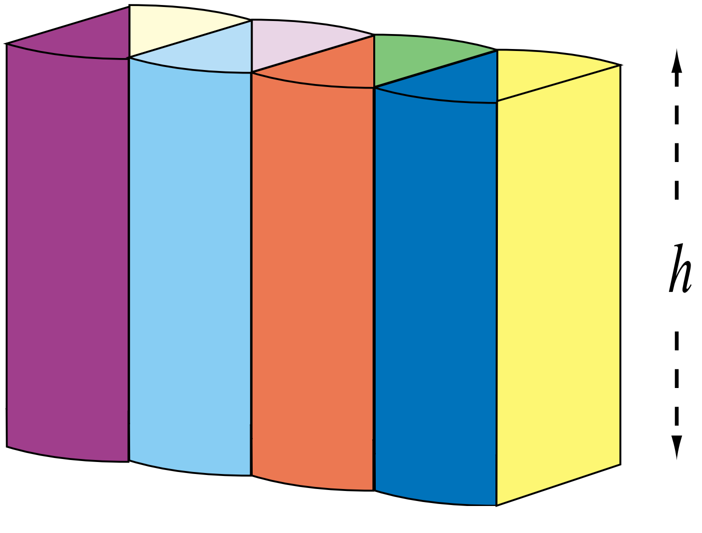


Fig 14(b)

Activity 15

Right circular cone

Materials required

chart paper,
a pair of scissors,
gum,
scale.

Objective

To make a cone of given slant length l and base circumference.

Pre-requisite knowledge

1. Circumference of a circle.
2. Sector of a circle.
3. Pythagoras theorem.

Procedure

1. Draw a circle with radius equal to the slant height l. [Fig 15(a)].
2. Mark a sector OAB such that arc length $A \times B$ equals the given base circumference of the cone. [Fig 15(a)].
3. Cut the sector AOB and gently fold, bringing the 2 radii OA and OB together. [Fig 15(b)]

Observations

1. Students will observe that when the radii of the sector are joined, a cone is formed.
2. The radius of the circle becomes the slant length of the cone.
3. The arc length becomes the circumference of the base of the cone.

Learning outcome

The students learn how to make a cone of given slant height and base circumference from a sector of a circle.

Remark

The teacher may ask student to determine the radius and the height of the cone formed, using the formula for the circumference of a circle and Pythagoras theorem.

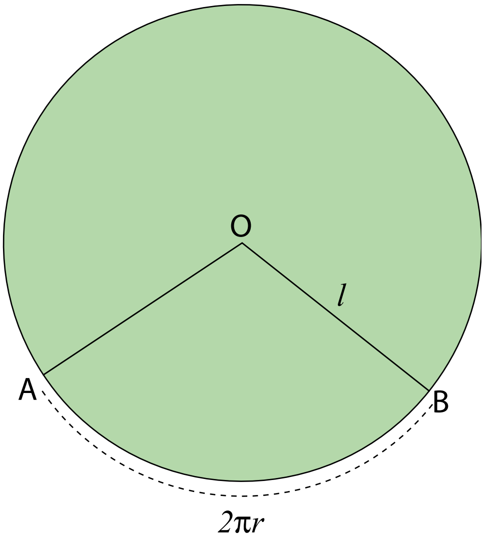


Fig 15(a)

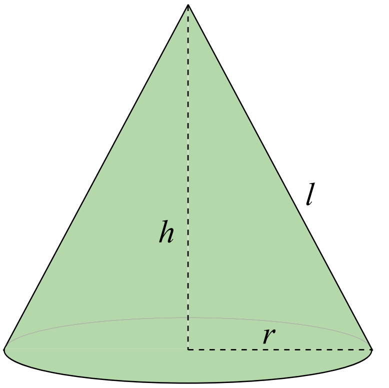


Fig 15(b)

Activity 16

Surface area of a cone

Materials required

chart papers,
a pair of scissors,
gum.

Objective

To give a suggestive demonstration of the formula for the lateral surface area of a cone.

Pre-requisite knowledge

1. The lateral surface of a cone can be formed from a sector of a circle.
2. Formula for area of a parallelogram.

Procedure

1. Cut vertically and unroll the cone. Identify the region. The region is a sector of circle. [Fig 16(a & b)]
2. Identify the arc length of the sector as the base circumference of the cone and the radius of the sector as the slant height of the cone.
3. Fold and cut the sector into 4 (even number of) equal smaller sectors. [Fig 16(b)]
4. Arrange the smaller sectors to form approximately a parallelogram. [Fig 16(c)]

Observations

1. Students observe that the base of the parallelogram is roughly half the circumference of the base of the cone. i.e. $\frac{1}{2} \times 2\pi r = \pi r$.
2. The height of the parallelogram is roughly the slant height of the cone 'l'.
3. Therefore, curved surface area = area of the parallelogram = πrl.

Learning outcome

1. Students learn that the surface area of a cone is πrl where r is the radius of the cone and l is the slant height.
2. Students appreciate how folding turns a plane surface (sector of a circle) into a curved surface (of the cone), and vice versa.

Remark

1. The teacher may help students observe that the base of the parallelogram is half the base circumference of the cone.
2. The teacher should point out that this activity does not give an exact proof of the formula, and the approximation improves by increasing the number of divisions of the sector.
3. The teacher may point out that the total surface area of the cone may be obtained by adding curved surface area to the area of the base.
   i.e. total area = $\pi rl + \pi r^2$

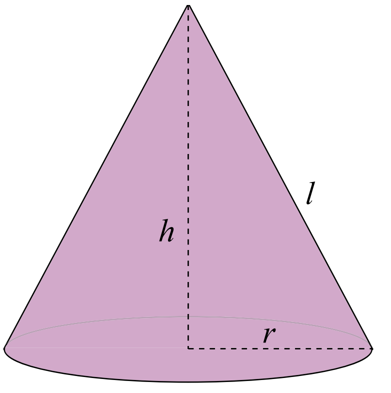


Fig 16(a)

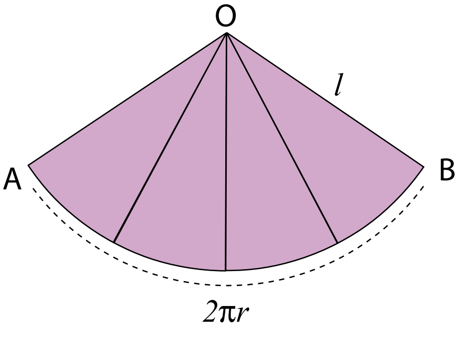


Fig 16(b)

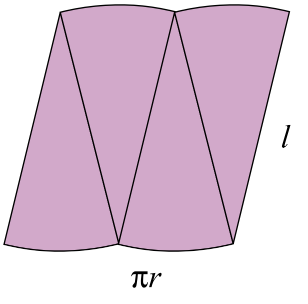


Fig 16(c)

Activity 17

Volume of a cone

Materials required

3 sets of a cone and cylinder. In each set, the cone and cylinder have the same height and base radius.

Objective

To give a suggestive demonstration of the formula for the volume of a right circular cone.

Pre-requisite knowledge

1. Concept of volume and its proportionality to quantity of matter.
2. Formula for the volume of a cylinder.

Procedure

1. Take one set of the cone and cylinder.
2. Fill the cone with sand.
3. Pour the sand from the cone to the cylinder.
4. Fill the cone again with sand and repeat step 3 to fill the cylinder completely with sand.
5. Repeat the activity with other sets of cones.

Observation

The student observes that for each set, it needs three pourings from the cone to fill the cylinder completely.

Learning outcomes

1. The volume of a cone is one-third the volume of the cylinder of the same height (h) and radius of base (r), i.e. equal to $= \frac{1}{3}\pi r^2 h$
2. Because of the simplicity of the concrete activity and the ratio (3) involved, students are likely to remember the result of the activity easily.

Remark

1. The teacher should see that the students fill the cone with sand properly.
2. The teacher should note that the activity makes use of the proportionality between volume and quantity of matter.

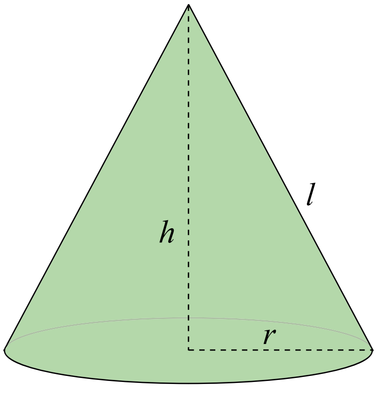


Fig 17(a)

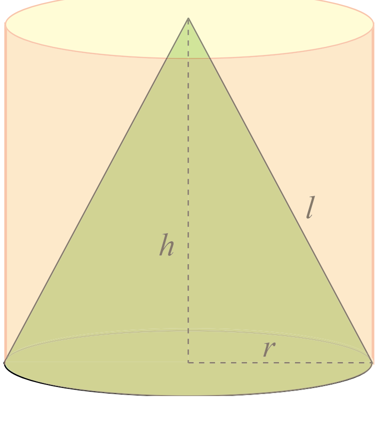


Fig 17(b)

Activity 18

Surface area of a sphere

Materials required

Hollow sphere cut into two hemispheres, a cylinder with both base diameter and height equal to the diameter of the sphere

Objective

To give a suggestive demonstration of the formula for the surface area of a sphere in terms of its radius.

Pre-requisite knowledge

Curved surface area of cylinder = $2\pi rh$

Procedure

1. Take a roll of a jute thread and wind it closely on the surface of the hemisphere completely. [Fig 18(a)].
2. Take another roll of jute thread and wind it completely along the curved surface of the cylinder. [Fig 18(b)].
3. Compare the length of the two threads.

Observations

1. Students observe that the length of the thread used to cover the curved surface of the cylinder is twice the length needed to cover the hemisphere.
2. Since the thickness of the thread is uniform and the same for both the threads, surface areas are proportional to the lengths of the threads approximately.
3. Hence surface area of the hemisphere = half the surface area of the cylinder

$$= \frac{1}{2} \times 2\pi rh$$

$$= \pi rh$$

$$= \pi r \times 2r \qquad (\because h = 2r)$$

$$= 2\pi r^2$$

Therefore, surface area of a sphere = $4\pi r^2$

Learning outcome

The student arrives at the formula for the surface area of a sphere through a simple activity, which relates it to the area of a cylinder.

Remark

The teacher should point out that this activity is not an exact proof of the formula, but is only a simple but approximate approach to appreciate the formula. The approximation improves with a thinner thread and tight and uniform winding.

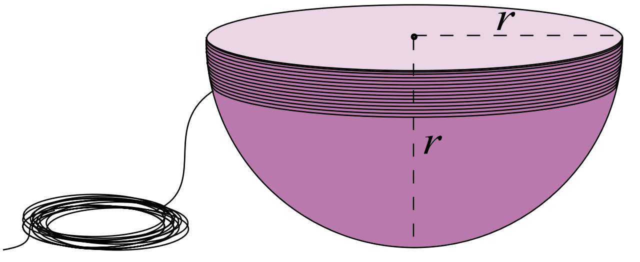


Fig 18(a)

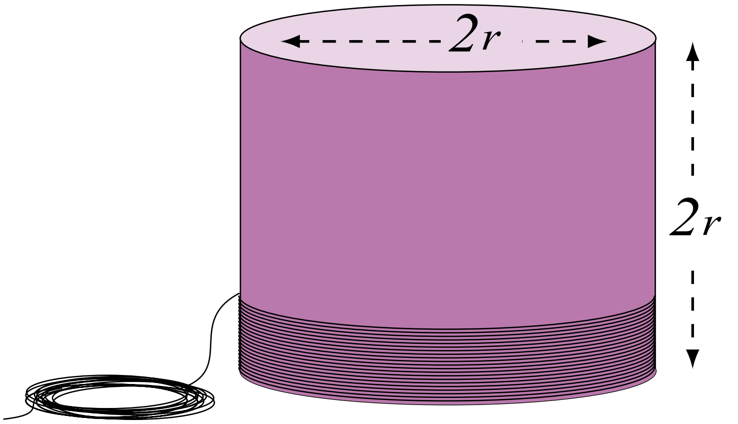


Fig 18(b)

Activity: 19

Volume of a sphere

Materials required

A hollow sphere and two cylinders whose base diameter and height are equal to the diameter of the sphere, sand.

Objective

To give a suggestive demonstration of the formula for the volume of a sphere in terms of its radius.

Pre-requisite knowledge

Volume of a cylinder.

Procedure

1. Fill the hollow sphere with sand once and empty it into one of the cylinders.
2. Fill the hollow sphere a second time with sand and empty it into the second cylinder.
3. Fill the hollow sphere a third time and empty it into the remaining spaces of the two cylinders.

Observations

1. Students observe that the total sand emptied in three pourings fill both the cylinders completely.
2. They, therefore, conclude that

   3 times the volume of sphere $= 2$ times the volume of cylinder $= 2\pi r^2 h$

   $= 4\pi r^3 \quad (\because h = 2r)$

   $\therefore$ Volume of sphere $= \dfrac{4}{3}\pi r^3$

Learning outcome

The students arrive at the formula for the volume of a sphere through a simple activity, which relates it to the volume of a cylinder.

Remarks

1. The teacher should point out that this activity is not an exact proof of the formula, but is only a simple but approximate approach to appreciate the formula. The approximation improves with the use of suitable materials (in place of sand) that do not leave air gaps.

2. The teacher can advise the students to try the activity with other suitable materials.

3. The teacher should note that the activity makes use of the proportionality between the volume and quantity of matter.

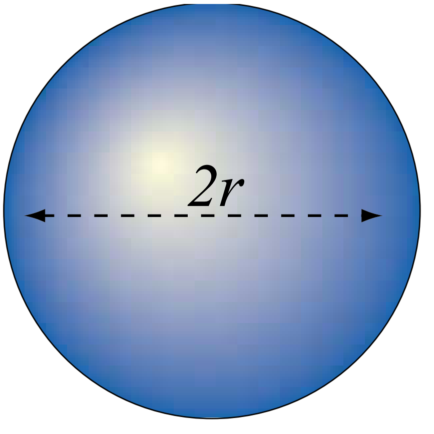


Fig 19(a)

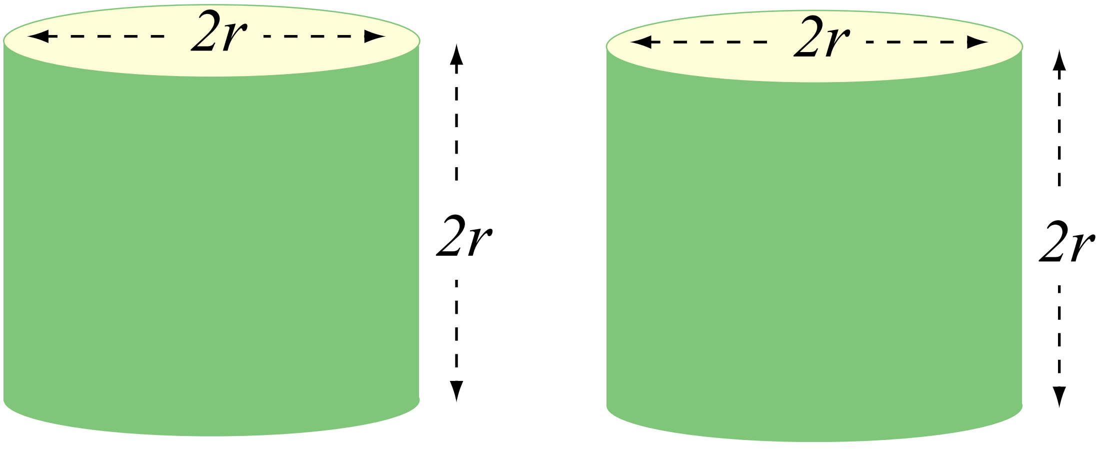


Fig 19(b)

Activity 20

Finding probability

Materials required

card board of size 15 cm × 15 cm, glazed paper (2 colours), pair of scissors, fevistick, sketch pens and an empty box.

Objective

To get familiar with the idea of probability of an event through a double colour card experiment.

Pre-requisite knowledge

The formula of probability of an event E is : P (E) = No. of favorable outcomes to E / Total no. of outcomes.

Procedure

A. Preparation of material for performing the activity.

1. Take a card board and paste glazed papers of different colours on both sides. (Say red and yellow.) [Fig 20(a)]
2. Cut the cardboard into 36 small squared cards.
3. Write all the 36 possible outcomes obtained by throwing two dice. [Fig 20(b)]. e.g. for the outcome (2, 1), write 2 on the yellow side and 1 on the red side of the squared card.
4. Put all the cards into a box.

B. For finding the required probability of an event do the following

1. Take out each card one by one without replacement and fill the observation table by putting (√) on favorable outcomes and (×) otherwise.
2. Count the total number of total possible outcomes from column 2. Write total possible outcomes.
3. Count the (√) marks from the columns 3, 4, 5 and 6.

Sr. No.	Possible outcomes		Sum ≥ 9	Sum < 5	Sum = 7	Odd on yellow & even on red.
	yellow card	red card				
1.	1	3	×	√	×	×
2.						
	....					
	....					
	....					
36.						

Observations

Total number of possible outcomes = ______________________________

Total number of favorable outcomes (Sum ≥ 9) = ____________________________

Total number of favorable outcomes (Sum < 5) = _______________________________

Total number of favorable outcomes (Sum = 7) = _______________________________

Total number of favorable outcomes (even number on one side of the card and odd on other) = _______________________________

Using the formula calculate the required probability of each event.

Remark

In this experiment, the student does not put back the card after taking it out. Consequently, the number of favourable outcomes for any event is certain. To arrive at the true notion of probability, the card should be put back and the experiment repeated a very large number of times. This, however, may be impractical in the actual classroom situation.

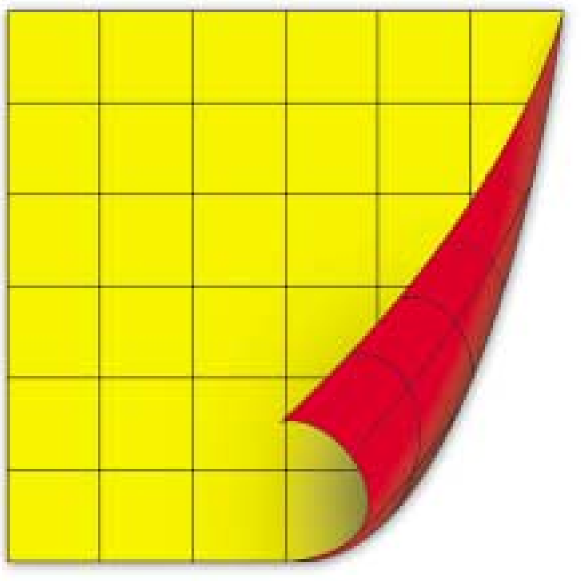


Fig 20(a)

(1,1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5,1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6,1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Fig 20(b)

Activity 21

Making a clinometer

Materials required

Stiff card, small pipe or drinking straw, thread, a weight (a metal washer is ideal)

Objective

To make a clinometer and use it to measure the height of an object.

Pre-requisite knowledge

1. Properties of right angled triangles.

Procedure:

1. Prepare a semi-circular protractor using any hard board and fix a viewing tube (straw or pipe) along the diameter.
2. Punch a hole (*o*) at the centre of the semicircle.
3. Suspend a weight {*w*} from a small nail fixed to the centre.
4. Ensure that the weight at the end of the string hangs below the protractor.
5. Mark degrees (in sexagecimal scale with 0^0 at the lowest and 1^0 to 90^0 proceeding both clockwise and anticlockwise). [Fig 21].

<u>Determining the height of an object</u>

6. First measure the distance of the object from you. Let the distance be *d*.
7. Look through the straw or pipe at the top of the object. Make sure you can clearly see the top of the object.
8. Hold the clinometer steady and let your partner record the angle the string makes on the scale of the clinometer. Let this angle be θ.

Using trigonometric ratio :

$$\tan\theta = \text{height} / \text{distance} = \frac{h}{d}$$

$h = d \times \tan\theta$

If, for example, $d = 100$ m and $\theta = 45^0$

$h = 100 \times \tan 45^0 = 100$ m

Learning outcome

Students learn how to determine the angle of elevation of an object and use it to determine the height of an object at a known distance.

Remark

Students may be asked to change the distance of the object (by either moving the object or by changing their position) and note how the angle of elevation varies. They will notice that though *d* and θ will vary, the product $h = d\tan\theta$ will be constant (within measurement error).

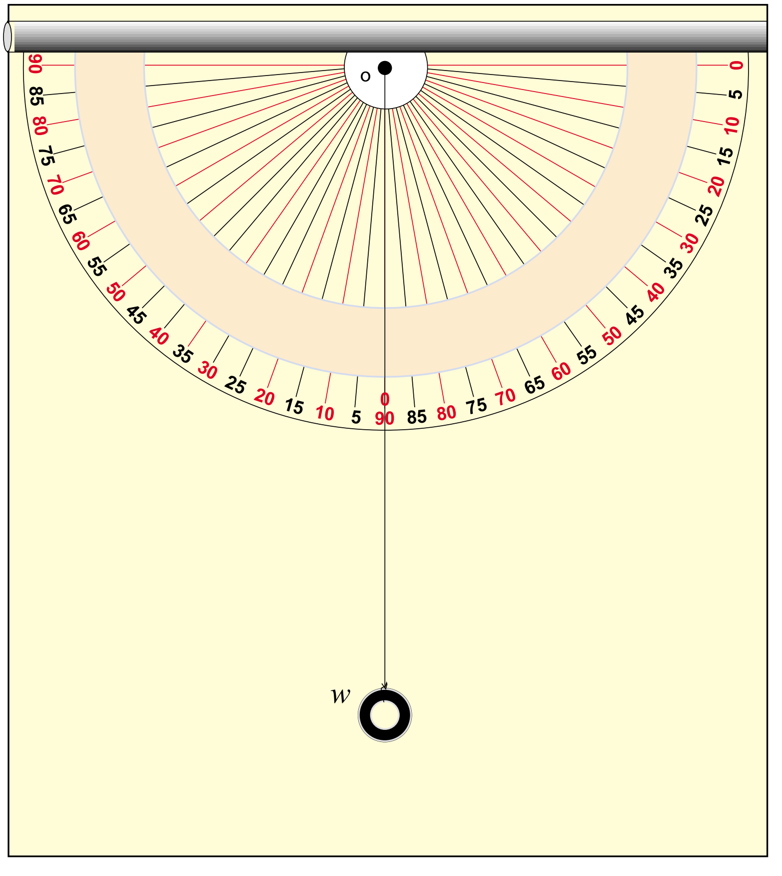


Fig 21

Project 1

Efficiency in packing

Objective

To investigate the efficiency of packing of objects of different shapes in a cuboid box. Efficiency is the percentage of box space occupied by the objects.

Description

1. Take a certain number of cylindrical tins and pack them in a cuboid container. For illustration let us take 81 tins.
2. The cylindrical tins can be placed in two different ways as shown in Fig P1(a) and Fig P1(b).
3. We wish to study which packing is more efficient.

Calculation

Square packing

Each base circle is circumscribed by a square.

Area of one circle = πR^2

Area of square = $4 R^2$

Area of circle / area of square = $\pi R^2 / 4 R^2$

This ratio will be evidently the same as the cross section of all the tins to the total base area.

Percentage efficiency $= \pi / 4 \times 100$

$= 78.5$ %

Hexagonal packing

Here we determine the sides of the base of the container in terms of the radius of the cylindrical tin. One side of the rectangular base i.e. BC = $18 \times R$.

To determine the other side, AB = $2 \times R + 9 \times h$, where h is the altitude of the equilateral triangle formed by joining the centres of three adjacent circles.

$h = 2R \sin 60^0$

$AB = 2R + 18 R \sin 60^0$

$\sin 60^0 = \sqrt{3} / 2$

So, $AB = 2R + 18R \times \sqrt{3} / 2$

$= (2 + 9\sqrt{3}) R$

Area of ABCD $= 18R \times (2 + 9\sqrt{3}) R$

$= 18R^2 (2 + 9\sqrt{3})$

Percentage efficiency $= 81\pi R^2 \times 100 / 18R^2 (2 + 9\sqrt{3})$

$= 80.3$ %

Remarks

1. In the calculation here the number of tins is fixed and the cuboid dimenssion is variable. A similar exercise may be done with fixed cuboid dimenssion and variable number of tins.
2. We can also determine the efficiency for packing of spheres in a cuboid.
   Volume of sphere = $4/3\ \pi R^3$
   Volume of cube = $8R^3$
   Percentage efficiency = $4\ \pi\ R^3 / 3 \times 8\ R^3 = \pi / 6$
   = 52 %

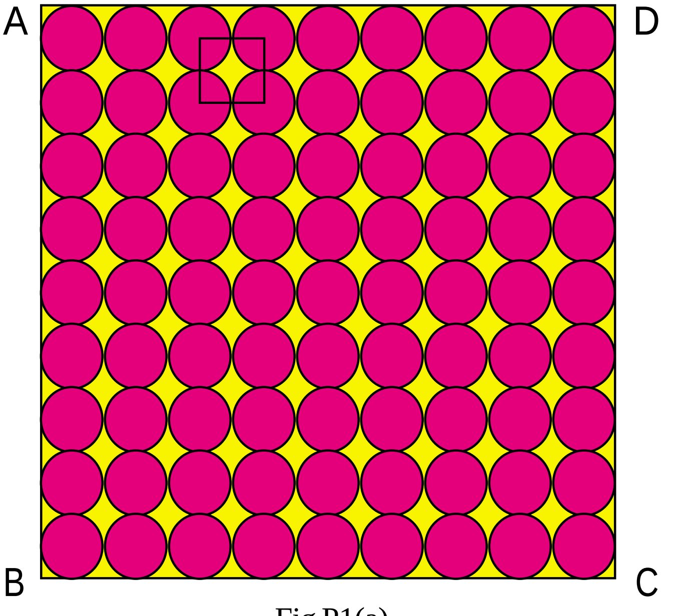


Fig P1(a)

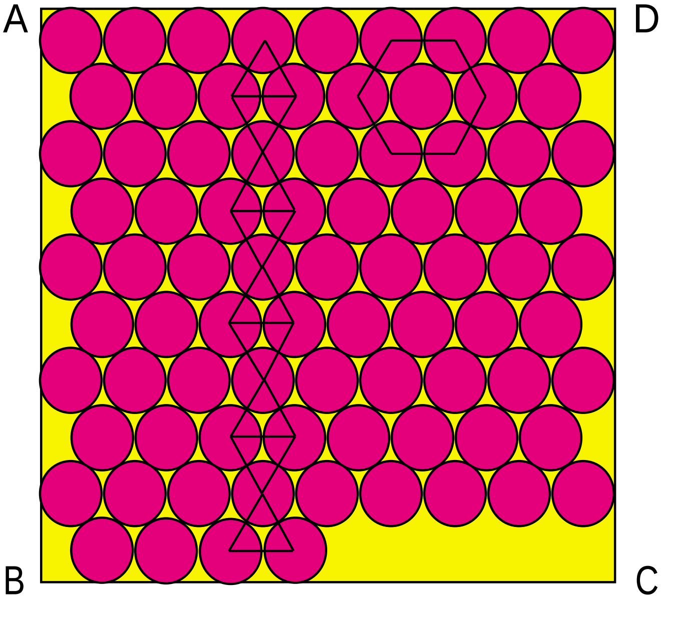


Fig P1(b)

Project 2

Geometry in real life

Description

In this project we try to find situations in daily life where geometrical notions can be effectively used. In particular, in the following examples the student discovers situations in which properties of similar triangles learnt in the classroom are useful.

1. How tall a mirror should you buy if you want to be able to see your full vertical image? We are given the fact that the angle of incidence equals the angle of reflection. Students will find that the mirror should be at least half his/her height.

2. To find the width of a pathway:

Fix a pole at Q directly opposite to a tree P on the other side of pathway.

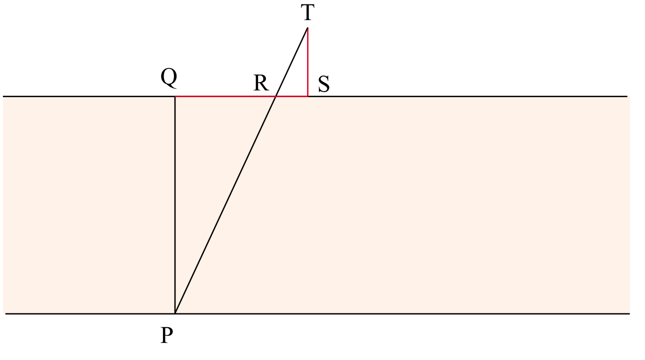


Fig P2(a)

Walk along the pathway, fix another pole at R at a known distance. Walk another known distance to S. From here, walk at right angles to the pathway till the point T is reached, such that T is directly in line with R and P. Measure the distance ST. Using the property of similarity of triangles, the width of the pathway is determined.

3. To find the height of a tree:

Place a ruler upright in the shadow of the tree, so that the end of its shadow is at the same place as the end of the shadow of the tree. Knowing the relevant distances, the height of the tree can be estimated.

As part of this project students should think of examples involving different geometrical properties of triangles and circles.

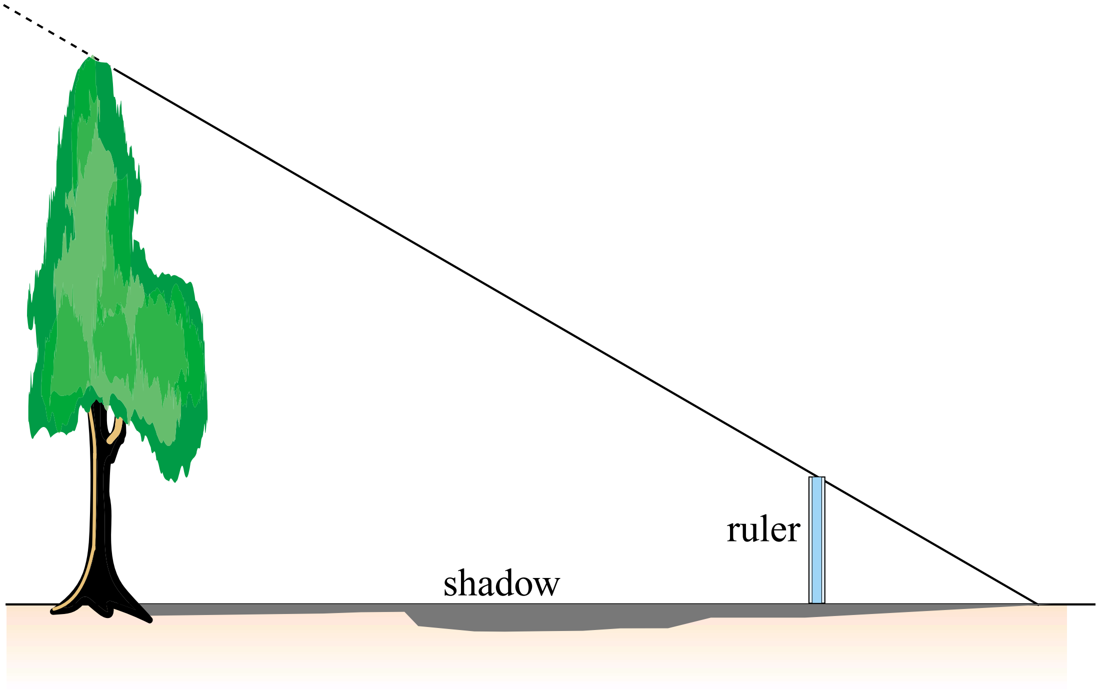


Fig P2(b)

Project 3

Experiment on Probability

Objective

To appreciate that finding probability through experiment is different from finding proability by calculation. Students become sesitive towards the fact that if they increase the number of observaions, probabilty found through experiment approaches the calculated probability.

Description

1. The teacher may ask the students to either work individually or at most in groups of two.
2. They will collect the following data by visiting any (say)10 classrooms in the school.

Class / section	No.of students	No. of children having birthdays in the month of											
		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
5A	48	6	5	8	3	2	1	3	6	3	5	2	4
6A													
7A													
8A													
9A													
10A													
Total													

3. They will obtain the fraction of number of children having their birthday in the month of January, February, … December from the data given in the table.

4. They will make a pie-diagram from the recorded data.

5. They will investigate if the fraction actually obtained in step 3 tallies with the calculated probability obtained for each month.
   e.g.:
   If total number of children whose birthday falls in the month of January is 38 and the total number of students is 500,
   the actual fraction of children born in January = 38/500
   Probability for a child to have birthday in January = 31/365

6. The students may increase their sample size, i.e. increase the number of observations and study if the actual fraction approaches the calculated probability. They should use a random sample for this purpose.

Project 4

Displacement and rotation of a geometrical figure

Objective

To study the distance between different points of a geometrical figure when it is displaced and / or rotated. Enhance familiarity with co-ordinate geometry.

Description

1. A cut-out of a geometrical figure such as a triangle is made and placed on a rectangular sheet of paper marked with x and y-axes.
2. The co-ordinates of the vertices of the triangle and its centroid are noted.
3. The triangular cut-out is displaced (along x-axis, along y-axis or along any other direction).
4. The new co-ordinates of the vertices and the centroid are noted again.
5. The procedure is repeated, this time by rotating the triangle as well as displacing it. The new co-ordinates of vertices and centroid are noted again.
6. Using the distance formula, distances between the vertices of the triangle are obtained for the triangle in original position and in various displaced and rotated positions.
7. Using the new co-ordinates of the vertices and the centroids, students will obtain the ratio in which the centroid divides the medians for various displaced and rotated positions of the triangle.

Result

Students will verify that under any displacement and rotation of a triangle the distances between vertices remain unchanged; also the centroid divides the medians in the ratio 2:1 in all cases.

Conclusion

In this project the students verify (by the method of co-ordinate geometry) what is obvious geometrically, namely that the length of sides of a triangle and the (relative) location of the centroid do not change when the triangle is displaced and / or rotated. The project will develop their familiarity with co-ordinates, distance formula and section formula of co-ordinate geometry.

Project 5

Frequency of letters / words in a language

Objective

Analysis of a language text, using graphical and pie chart techniques.

Description

1. The teacher may ask the students to work individually or in groups of two.
2. Students will select any paragraph containing approximately 250 words from any source. e.g. newspaper, magazine, textbook, etc.
3. They will read every word and obtain a frequency table for each letter of the alphabet as follows

Letter	Tally marks	Frequency
A		
B		
C		
.		
.		
.		
.		
Z		

4. They will note down the number of two-letter words, three-letter words, …. so on and obtain a frequency table as follows

Number of words with	Tally marks	Frequency
2 letters		
3 letters		
.		
.		
.		
.		

5. Select 10 different words from the text which have frequency greater than 1. Give ranks 1, 2, 3, …., 10 in decreasing order of their frequency. Obtain a table as follows

Selected word	Frequency	Rank
e.g. on		
it		
.		
.		
.		
.		

6. Investigate the following

From table 1

a) What is the most frequently occurring letter?
b) What is the least frequently occurring letter?
c) Compare the frequency of vowels
d) Which vowel is most commonly used?
e) Which vowel has the least frequency?
f) Make a pie chart of the vowels a, e, i, o, u, and remaining letters. (The pie chart will thus have 6 sectors.)
g) Compare the percentage of vowels with that of consonants in the given text.

From table 2

a) Compare the frequency of two letter words, three letter words, ….. and so on.
b) Make a pie chart. Note any interesting patterns.

From table 3

a) The relation between the frequency of a word to its rank.
b) Plot a graph between the frequency and reciprocal of word rank. What do you observe? Do you see any interesting pattern?
c) Repeat the experiment by choosing text from any other language that you know and see if any common pattern emerges.

Group activity 1

Fourth order Magic Dance

The interplay of mathematics and art can be very appealing. Here is an attempt to present a versatile form of the fourth order magic square through a dance.

Step 1

1. A group of 16 dancers present themselves on the stage in 2 rows of 8 each.
2. The first row faces the audience and carry flash cards to show fixed numbers 1 to 8 right to left.
3. The second row of dancers squat at the rear, facing back stage. Each one has 2 sets of flash cards, each set showing numbers 0 to 9 to get a two digit number.

Step 2

1. Any even number from 34 to 98 is called out. This is considered as the magic square number for building the magic square through dance. Suppose the number taken is 62.

Step 3

Music begins. Dance begins for the first row. The dancers in first row carry FIXED NUMBERS. While they do so, dancers in the rear are busy calculating and fixing the numbers they have to hold in their hands. Once the numbers are calculated they join the dance, keeping their palms in folded positions. Their calculations are as shown below. They take 30 seconds to calculate their numbers on their palms.

REAR ROW (8 dancers)	$\frac{62}{2} - 1 = 30$	$\frac{62}{2} - 2 = 29$	$\frac{62}{2} - 3 = 28$	$\frac{62}{2} - 4 = 27$	$\frac{62}{2} - 5 = 26$	$\frac{62}{2} - 6 = 25$	$\frac{62}{2} - 7 = 24$	$\frac{62}{2} - 8 = 23$
FRONT ROW (8 dancers)	8	7	6	5	4	3	2	1

Note: Use a rubber band to hold the card on the palm. The cards are chosen by the dancers after calculation. They face the audience now holding their respective flash cards with their numbers as below.

Step 4

1. It is a fourth order magic square, so there are 16 positions.
2. Dancers holding numbers 1 to 8 have fixed positions in this formation as shown in the table.
3. Dancers holding numbers 23 to 30 also have their positions predetermined - as shown in the table.

	Column 1	Column 2	Column 3	Column 4
Row 4	$\frac{62}{2} - 8 = 23$	$\frac{62}{2} - 1 = 30$	2	7
Row 3	6	3	$\frac{62}{2} - 4 = 27$	$\frac{62}{2} - 5 = 26$
Row 2	$\frac{62}{2} - 2 = 29$	$\frac{62}{2} - 7 = 24$	8	1
Row 1	4	5	$\frac{62}{2} - 6 = 25$	$\frac{62}{2} - 3 = 28$

This formation is seen to present the required magic square for the magic sum 62.

Now
1. A display of row-by-row presentation is made by the dancers.
2. A display of column-by-column presentation is made.
3. A display of diagonal presentation is made.

The audiences check the total and appreciate the dancers.
Finally, the 16 dancers leave the stage gracefully.

Rationale

Mathematically, it is easily seen that this pattern of fixing the positions gets a versatile magic square. The magic sum has been taken to be even. Let it be $2n$. See the table below. It can be seen that the rows, columns and diagonals each add upto $2n$.

$n - 8$	$n - 1$	2	7
6	3	$n - 4$	$n - 5$
$n - 2$	$n - 7$	8	1
4	5	$n - 6$	$n - 3$

This can be performed for any other even number from 34 to 98.

Group activity 2

Live Lattice

Live lattice is a lattice formed by students placed in square or rectangular formation .

Objective

Using the row number and column number (of the students standing in a rectangular array) establish the following algebraic concepts through demonstration:

1. meaning of ordered pair: (x, y) is not equal to (y, x) unless $x = y$.
2. equations of straight lines, parallel lines and intersecting lines.
3. simultaneous equations.
4. inequations.

Procedures

1. Ask students to stand in a suitable rectangular array.
2. Each student represents an ordered pair. For example a student standing in 2^{nd} row and 3^{rd} column stands for (3, 2) ie (c,r). 'c' stands for column number and 'r' for row number. [Fig G2(a)]

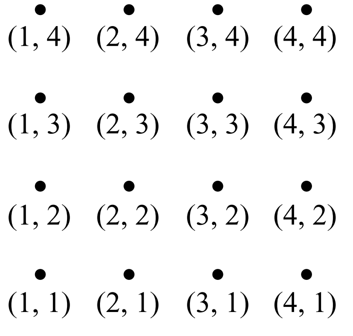


Fig G2(a)

3. Students will write their column number on their left palm and row number on their right plam. (Tie cards on their palms using rubber bands.)
4. Give commands to the students to understand the mentioned algebraic concepts.

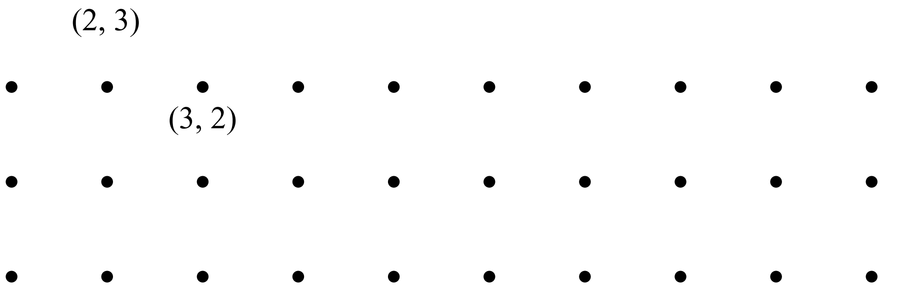


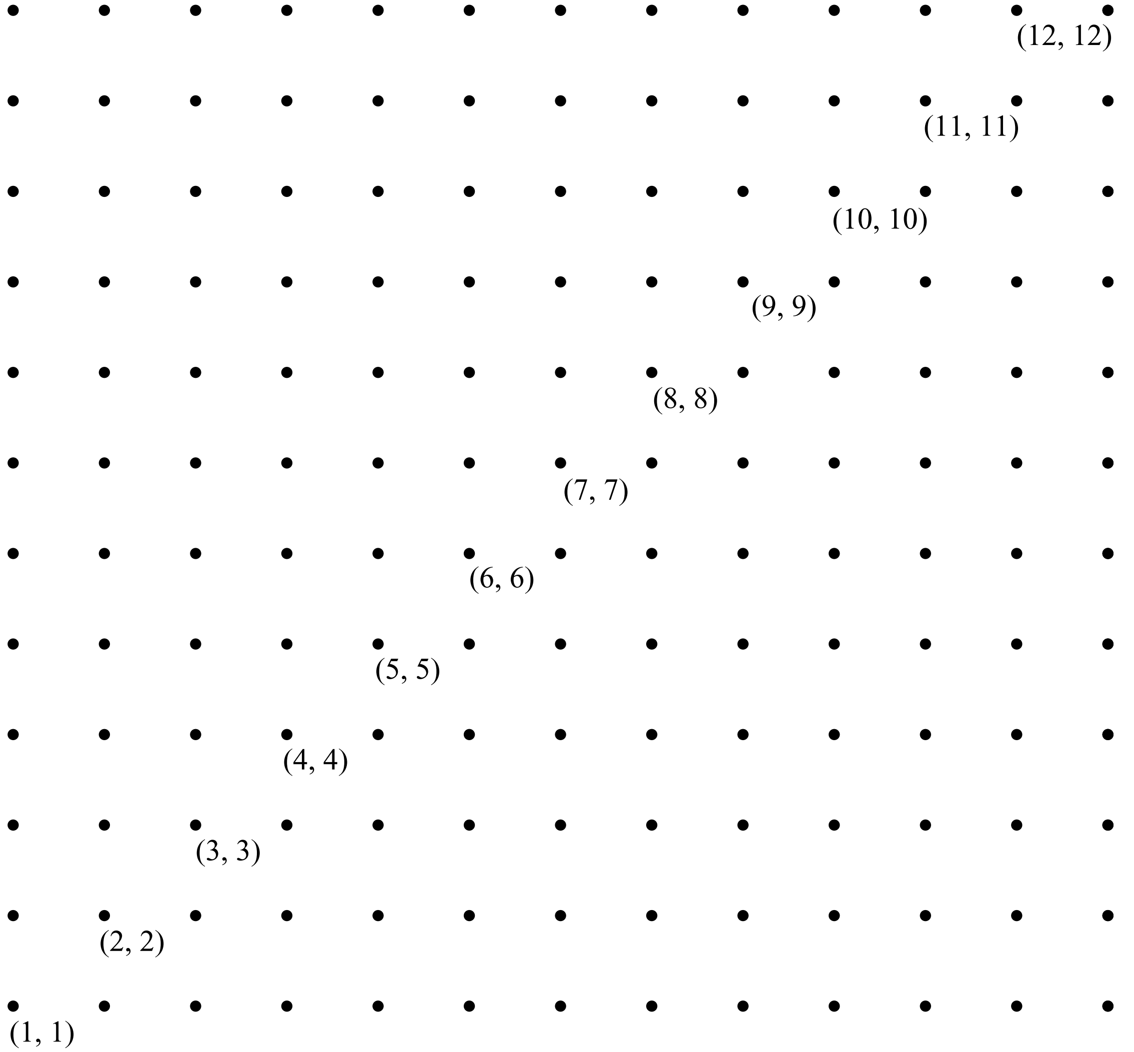
Fig G2 (b)

Commands

Ask students to check the following places.

1. (2, 3) & (3, 2)
2. (5, 2) & (2, 5)
3. (1, 2) & (2, 1)

What do they observe? Record the findings.



FigG2(c)

Command

Ask students whose column number and row number are same, ie. c=r , to stand up on their places.

What do they observe? Record the findings.

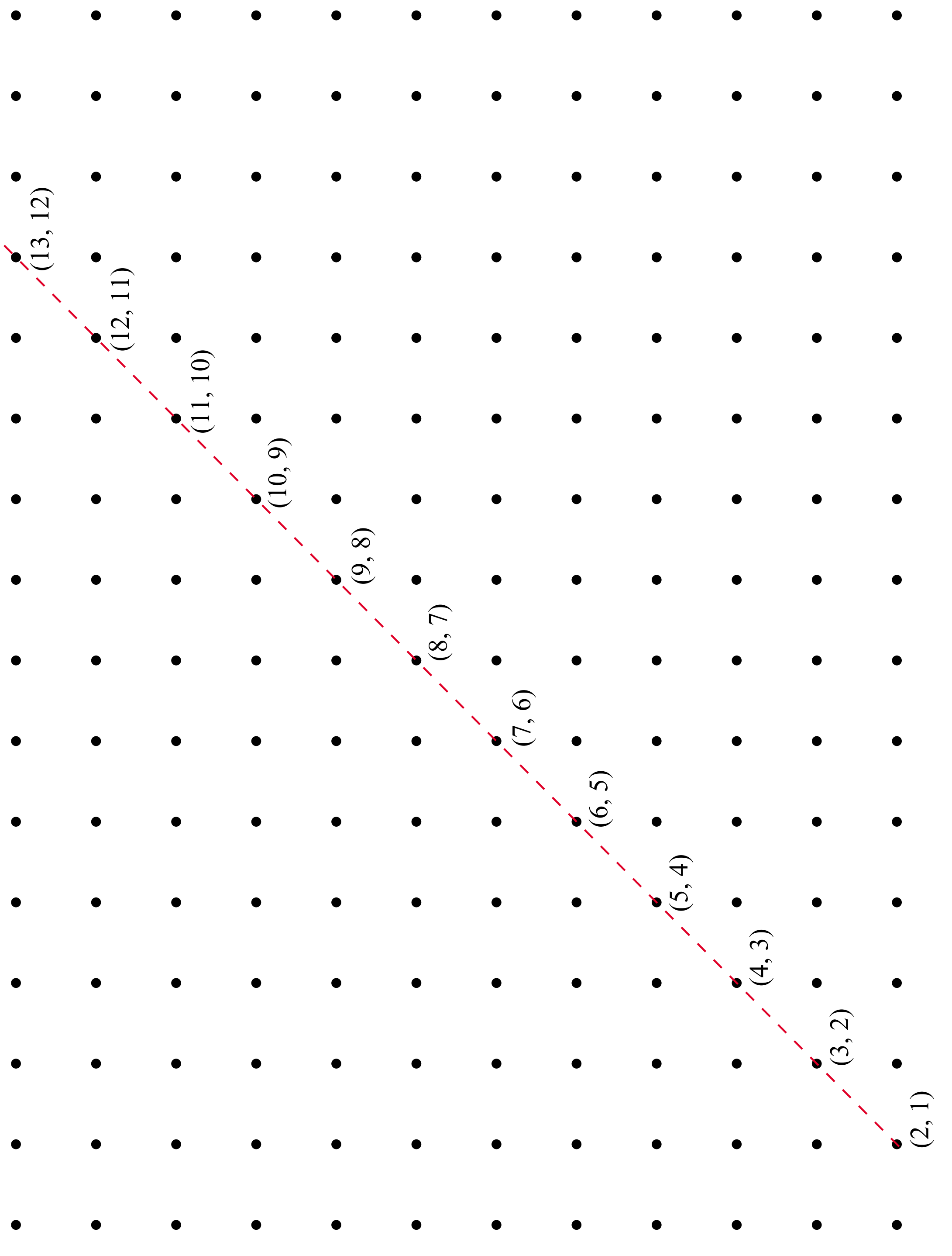


Fig G2(d)

<u>*Command*</u>

Ask students whose column number and row number is 1 more than their row number, i.e. c = r+1 , to stand up on their places.

i.e. students of ordered pair (r+1, r)]

What do they observe? Record the findings.

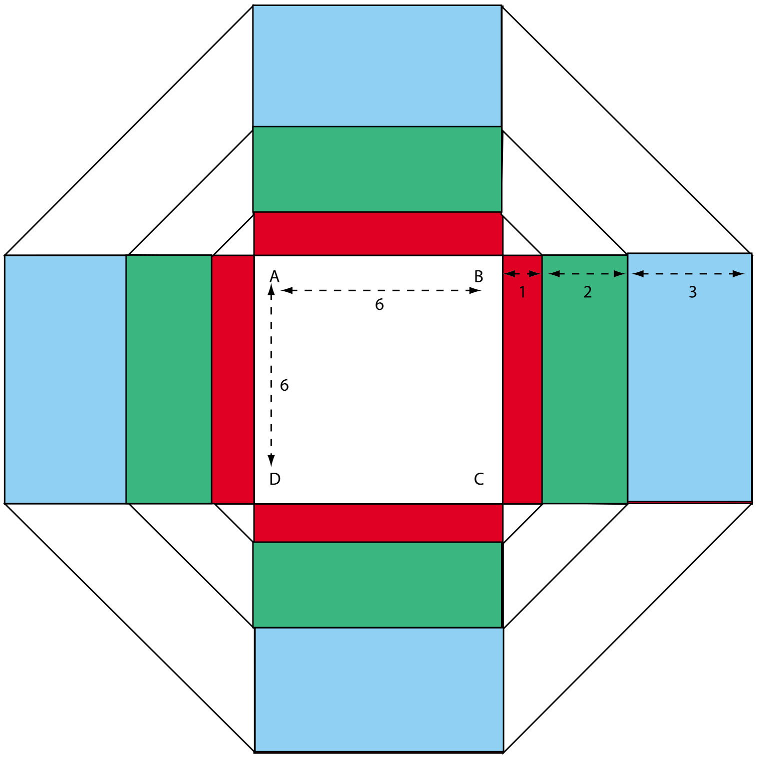
Other concepts can be handled similarly.

Suggested Project 1

Mathematical designs and patterns using arithmetic progression.

Brief description

In this project the students may work individually or in groups of two. They will explore mathematical designs and patterns using the notion of arithmetic progression. They will explore the areas where such designing techniques can be applied.



Example

1. Take terms in an arithmetic progression say a_1, a_2, a_3.
2. First of all, the starting base shape of the design will be prepared from a paper. For that add the given numbers i.e. $s = a_1 + a_2 + a_3$. Cut a square (of side s cm) and paste it on a sheet.
3. Cut rectangular strips of different colours of size $a_1 \times s$, $a_2 \times s$, $a_3 \times s$. (Note that a_1, a_2, a_3 are in arithmetic progression.)
4. Paste the rectangular strips adjacent to each other on one side of the square.
5. Cut similar rectangular strips for the remaining 3 sides.
6. Join the gap by straight lines. Right angled triangles are formed. This is how one such design can be easily made where the rectangular strip's width is changing in an arithmetic progression and the length is the same.
7. You may take any other regular polygon as the starting shape and build newer designs.
8. You may take a circle as the starting shape, take equidistant points on its circumference say 1, 2, …., 36, and repeat similar procedure.

Suggested Project 2

Early History of Mathematics

Description

This project is meant to develop the student's awareness of the history of mathematics. The student should give an outline of the major milestones in mathematics from Euclid to say Euler.

Suggested Project 3

Analysis of test results and interpretation.

Description

After the half yearly or annual examination, the marks of the students may be tabulated as follows:
(Take the size of class interval = 5 preferably)

Range of Marks	Tally Marks	Frequency

Now, present the data in the form of a histogram and a pie chart. This tabulation can be done for marks in individual subjects as well as for aggregate marks.
Interpret the data in different ways (e.g. how many children need special guidance in say mathematics, etc.)

Acknowledgements

Prof. Arvind Kumar

Ms. K. Alamelu

Ms. Rashmi Kathuria

Mr. Sai Baba

Ms. Shweta S. Naik

Ms. Jharne De

Mr. D.R. Sharma

CBSE Faculty
Shri G. Balasubramanian
Director (Academic)

Shri R.P. Sharma
Education Officer (Science)
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