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Test Booklet Series

D

TEST BOOKLET
MATHEMATICS

Time Allowed : Two Hours

PAPER-III

Maximum Marks : 200

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS BOOKLET DOES NOT HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.

2. **Please note that it is the candidate's responsibility to encode and fill in the Roll number and Test Booklet Series Code A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR answer sheet. Any omission/discrepancy will render the answer sheet liable for rejection.**

3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. DO NOT write *anything else* on the Test Booklet.

4. This Test Booklet contains 100 items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.

5. You have to mark all your responses ONLY on the separate Answer Sheet provided. See directions in the Answer Sheet.

6. All items carry equal marks.

7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.

8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator *only the Answer Sheet.* You are permitted to take away with you the Test Booklet.

9. Sheet for rough work are appended in the Test Booklet at the end.

10. **Penalty for wrong answers :**

THERE WILL BE PENALTY FOR WRONG ANSWER MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.

(i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third (0.33)** of the marks assigned to that question will be deducted as penalty.

(ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answer happens to be correct and there will be same penalty as above to that question.

(iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that questions.

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1. Let f(x) = $\begin{cases} x^2 & x \in Z \\ \dfrac{k(x^2-4)}{2-x} & x \notin Z \end{cases}$ where Z is the set of all integers. Then f(x) is continuous for

(a) k = 1 only (b) every real k
(c) every real k except k = –1 (d) k = –1 only

Sol. **(d)**

2. What is the area enclosed by the curve $y^2 = -x\,|x|$ and $x = -1$?
(a) 1/2 square units (b) 1 square unit
(c) 2 square units (d) None of the above

Sol. **(b)**

$y^2 = x|x| = \begin{cases} -x^2 & x \geq 0 \\ x^2 & x < 0 \end{cases}$ (Not possible)

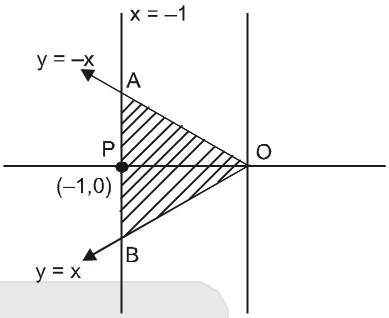
$\Rightarrow \quad y^2 - x^2 = 0$

$\Rightarrow \quad (y + x)(y - x) = 0 \qquad x < 0$

The area of OAB is

$= \dfrac{1}{2}(AB)(OP)$

$= \dfrac{1}{2}(2)(1) = 1$ sq unit.



3. If $I_1 = \displaystyle\int_e^{e^2} \frac{dx}{\ln x}$ and $I_2 = \displaystyle\int_1^2 \frac{e^x dx}{x}$ then which one of the following is correct ?

(a) $2I_1 = I_2$ (b) $I_1 = 2I_2$ (c) $I_1 = I_2$ (d) $I_1 = e\,I_2$

Sol. **(c)**

$I_1 = \displaystyle\int_e^{e^2} \frac{dx}{\ln x}$

$I_2 = \displaystyle\int_1^2 \frac{e^x dx}{x}$

Put $e^x = t \Rightarrow x = \ln t \Rightarrow dx = \dfrac{1}{t}\,dt$

$\Rightarrow \quad I_2 = \displaystyle\int_e^{e^2} \frac{dt}{\ln t} = \int_e^{e^2} \frac{dx}{\ln x} = I_1$

4. If $I_r = \displaystyle\int_0^{\frac{\pi}{4}} \tan^r x\,dx$, then $I_2 + I_4$, $I_3 + I_5$, $I_4 + I_6$ ... are in

(a) AP (b) GP (c) HP (d) None of the above

Sol. **(c)**

$$I_r = \int_0^{\frac{\pi}{4}} \tan^r x \, dx$$

$$I_{r+2} + I_r = \int_0^{\frac{\pi}{4}} (\tan^{r+2} x + \tan^r x) dx$$

$$= \int_0^{\frac{\pi}{4}} \tan^r x \sec^2 x dx$$

$$= \left. \frac{\tan^{r+1} x}{r+1} \right|_0^{\pi/4} = \frac{1}{r+1}$$

$I_2 + I_4$, = $\frac{1}{3}$; $I_3 + I_5$, = $\frac{1}{4}$, $I_4 + I_6$, = $\frac{1}{5}$ are in H.P.

Directions : For the next two (02) questions that follow : Let f(x) = $\sum_{k=0}^{n} a_k |k-1|^k$ where $a_k \in R$

5. Consider the following statements

1. f(x) is continuous function for all $a_k \in R$.
2. f(x) is differentiable function for all $a_k \in R$.

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

Sol. **(a)**

$f(x) = a_0 + a_1(x-1) + a_2(x-1)^2 + a_3(x-1)^3 + ... + a_n|x-1|^n$ as we know that modulus function is continuous for all $x \in R$.

Also f(x) is not diff. at x = 1 when $a_1 \neq 0$.

6. Consider the following statements :

1. f(x) is continuous at x = 1 provided $a_{2k} = 0$
2. f(x) is differentiable at x = 1 provided $a_{2k+1} = 0$

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

Sol. **(b)**

f(x) is continuous $\forall\, a_k \in R$ so statement-1 is false.

also $a_{2k+}1 = 0$ the $f(x) = a_0 + a_2(x-1)^2 + a_4(x-1)^4 + ...$ which is differentiable at x = 1

$\Rightarrow$ Statement – 2 is true.

7. What is $\int_{\frac{3\pi}{4}}^{\frac{4\pi}{3}} \frac{\cos x . \sin x}{|\cos x|} dx$?

(a) $\frac{\sqrt{2}-1}{\sqrt{2}}$ (b) $\frac{1+\sqrt{2}}{2}$ (c) $\frac{\sqrt{2}-1}{2}$ (d) None of the above

Sol. **(c)**

$$\int_{\frac{3\pi}{4}}^{\frac{4\pi}{3}} \frac{\cos x.\sin x}{|\cos x|}\,dx$$

$$= \int_{\frac{3\pi}{4}}^{\frac{4\pi}{3}} -\sin x\,dx \qquad (\because\ |\cos x| = -\cos x)$$

$$= \cos x\Big|_{3\pi/4}^{4\pi/3} = \cos\frac{4\pi}{3} - \cos\frac{3\pi}{4} = -\frac{1}{2} + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{2}.$$

Directions : For the next two (02) questions that follow :

Consider the function $f(x) = |x^2 - 2|$ on the interval $-3 < x < 3$.

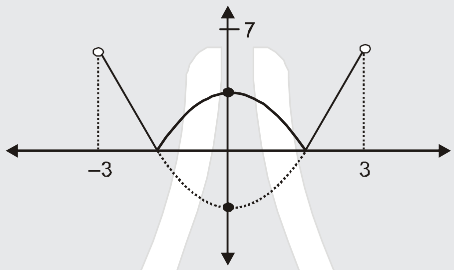
8. Consider the following statements :

1. The functions attains relative minimum at only one point.
2. The function attains relative maximum at only one point.

Which of the above statements/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Sol. **(b)**



$f(x) = |x^2 - 2|$ by the graph.
It is obviously true that f(x) attains relative maximum at only one point and relative minimum at two points.

9. Consider the following statements :

1. The absolute maximum value of the function is 2.
2. The absolute minimum value of the function is 0.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Sol. **(b)**

The graph also absolute minimum value of the function is 0
but absolute maximum is 2 not true.

10. If [x] denotes the greatest integer less than or equal to x, then what is $\lim\limits_{n\to\infty} \frac{[x]+[2x]+[3x]+...[nx]}{n^2}$ equal to?

(a) x (b) x/2 (c) x/3 (d) 0

Sol. $\lim\limits_{n\to\infty} \frac{[x]+[2x]+[3x]+.......+[nx]}{n^2}$

$\because$ $rx \le [rx] < rx + 1$

$\Rightarrow$ $\sum\limits_{r=1}^{n} rx = \sum\limits_{r=1}^{n} [rx] < \sum\limits_{r=1}^{n} [rx+1]$

$\Rightarrow \quad \frac{n(n+1)}{2}x \le [x] + [2x] + [3x] + ... + [nx] < \frac{n(n+1)}{2}x + n.$

$$\Rightarrow \quad \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}x}{n^2} \le \lim_{n\to\infty} \frac{[x]+[2x]+...+[nx]}{n^2} < \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}x+n}{n^2}$$

$$\Rightarrow \quad \frac{1}{2}x \le \lim_{n\to\infty} \frac{[x]+[2x]+....+[nx]}{n^2} < \frac{1}{2}x.$$

$$\Rightarrow \quad \lim_{n\to\infty} \frac{[x]+[2x]+...+[nx]}{n^2} = \frac{1}{2}x.$$

Directions : For the next two (02) questions that follow :

A line L is perpendicular to the lines having direction ratios <3, 2, 1> and <1, 2, 3>.

11. What is the sum of the direction cosines of the line L?

(a) 0 (b) 1 (c) 3 (d) None of the above

Sol. **(a)**

Let D' Ratios of line L are <a, b, c>

$\Rightarrow \quad 3a + 2b + c = 0 \quad ...(1)$

and $\quad a + 2b + 3c = 0 \quad ...(2)$

$$\Rightarrow \quad \frac{a}{4} = \frac{b}{-8} = \frac{c}{4}$$

$$\Rightarrow \quad \frac{a}{1} = \frac{b}{-2} = \frac{c}{1}$$

$\Rightarrow$ Sum of D'cosines of Line L = 0.

12. Consider the following statements :

1. The sum of the direction ratios of the line L is equal to sum of the direction cosines of the line L.
2. The angle made by the line L with the y-axis is twice the angle made by the line L with the x-axis.

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

Sol. **(d)**

Obviously statement – 1 is false.

also direction cosine of the line are $< \pm\frac{1}{\sqrt{6}}, \pm\frac{2}{\sqrt{6}}, \pm\frac{1}{\sqrt{6}} >$

$\Rightarrow \quad \cos\beta = \mp\frac{2}{\sqrt{6}}$ and $\cos\alpha = \mp\frac{1}{\sqrt{6}}$

$\Rightarrow \quad \cos2\alpha = 2\cos^2\alpha - 1$

$= \frac{2}{6} - 1 = -\frac{4}{6} \ne \cos\beta$

$\Rightarrow$ Statement – 2 is also false.

Directions : For the next three (03) questions that follow : Consider the plane containing the line $\frac{x+1}{-3} + \frac{y-3}{2} = \frac{z+2}{-1}$ and passing through the point (1, –1, 0).

13. What are the direction ratios of the normal to the plane ?

(a) <1, 1, 1> (b) <1, –1, 1> (c) <1, –1, 1> (d) None of the above

14. What is the angle made by the plane with the x-axis?

(a) $\tan^{-1}\sqrt{2}$ (b) $\cot^{-1}\sqrt{2}$

(c) $\frac{\pi}{6}$ (d) None of the above

15. What is the question of the plane ?

(a) x + y – z = 0 (b) –3x + 2y – z + 6 = 0

(c) x + y + z = 0 (d) None of the above

Sol. **13.** **(d)**

14. **(d)**

15. **(d)**

Equation of plane containing the line $\frac{x+1}{-3} = \frac{y-1}{2} = \frac{z+1}{-1}$ and passes through the point (1, –1, 0) is

$$\begin{vmatrix} x-1 & y+1 & z-0 \\ -2 & +4 & -2 \\ -3 & 2 & -1 \end{vmatrix} = 0$$

$\Rightarrow$ (–4 + 4) (x –1) – (2 – 6) (y + 1) + (–4 + 12) (z – x) = 0

$\Rightarrow$ 4(y + 1) + 8(z) = 0

$\Rightarrow$ y + 2z + 1 = 0.

D. Ratio's of the normal are <0, 1, 2>

Angle of the plane with x-axis is 90°

equation of plane y + 2z + 1 = 0.

16. If A = {1, 2, 3} and B = {1, 2} and C = {4, 5, 6}, then what is the number of elements in the set $A \times B \times C$?

(a) 8 (b) 9 (c) 15 (d) 18

Sol. **(d)**

n(A) = 3, n(B) = 2; n(C) = 3

$\Rightarrow$ $n(A \times B \times C)$ = 3.2.3. = 18.

17. There are unlimited number of identical balls of four different colours. How many arrangements of at most 6 balls can be made by using them ? [to be arranged in a single row only]

(a) 1365 (b) 2730 (c) 5460 (d) None of the above

Sol. **(c)**

18. Let Z be the set of integers and ‘o’ be a binary operation of Z defined by a o b = a + b – ab for all, a, b $\in$ Z. What is the inverse of an element a ($\neq$ 1) $\in$ Z ?

(a) a/(a –1) (b) a/(1 – a) (c) (a – 1)/a (d) None of the above

Sol. **(a)**

Let Identity element be e

$\Rightarrow$ aoe = a + e – ae = a = eoa $\forall$ a $\in$ z

$\Rightarrow$ e(1 – a) = 0

$\Rightarrow$ e = 0

Now let inverse of a is a^{-1} then

$a o a^{-1} = e = 0$

$\Rightarrow$ $a + a^{-1} - a.a^{-1} = 0$

$\Rightarrow$ $a^{-1}(1 - a) = -a$

$\Rightarrow$ $a^{-1} = \left(\frac{a}{a-1}\right)$.

19. If $x^2 - x + 1 = 0$, then what is the value of $\sum_{n=1}^{5}\left(x^n + \frac{1}{x^n}\right)^2$?

(a) 8 (b) 10 (c) 12 (d) None of the above

Sol. **(a)**

$x^2 - x + 1 = 0$

$\Rightarrow \quad x = \frac{1 \pm \sqrt{3}i}{2}$

$\Rightarrow \quad x = -\omega, -\omega^2$

$$\Rightarrow \quad \sum_{n=1}^{5}\left(x^n + \frac{1}{x^n}\right)^2 = \left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \left(x^4 + \frac{1}{x^4}\right)^2 + \left(x^5 + \frac{1}{x^5}\right)^2$$

$= 1 + 1 + 4 + 1 + 1 = 8.$

20. If n is the positive integer such that $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, then what is the value of $\sum_{r=0}^{2n} a_r$?

(a) 0 (b) 3^{n+1} (c) 3^{n-1} (d) 3^n

Sol. **(d)**

$(1 + x + x^2)^n = a_0 + a_1 + a_2x^2 + ... + a_{2n}\, x^{2n}.$

Put $x = 1 \Rightarrow a_0 + a_1 + a_2 + ... + a_{2n} = 3^n.$

21. If $(x + 1)(x + 2)(x + 3)(x + 6) = 3x^2$, then the equation has

(a) all imaginary roots (b) all real roots

(c) two rational and two irrational roots (d) two imaginary and two irrational roots

Sol. **(d)**

$(x + 1)(x + 2)(x + 3)(x + 6) = 3x^2$

$\Rightarrow \quad (x^2 + 7x + 6)(x^2 + 5x + 6) = 3x^2$

$\Rightarrow \quad \left(x + \frac{6}{x} + 7\right)\left(x + \frac{6}{x} + 5\right) = 3$

Let $\left(x + \frac{6}{x}\right) = t$

$\Rightarrow \quad (t + 7)(t + 5) = 3$

$\Rightarrow \quad t^2 + 12t + 32 = 0$

$\Rightarrow \quad (t + 4)(t + 8) = 0$

$\Rightarrow \quad t = -4, -8$

when $x + \frac{6}{x} = -4 \quad \Rightarrow \quad x^2 + 4x + 6 = 0$

$\Rightarrow$ Imaginary roots.

when $x + \frac{6}{x} = -8 \quad \Rightarrow \quad x^2 + 8x + 6 = 0$

$\Rightarrow \quad x = \frac{-8 \pm 2\sqrt{10}}{2}$

$\Rightarrow \quad x = -4 \pm \sqrt{10}$

Hence two imaginary and two irrational roots.

22. If $z = \frac{1+i}{1-i}$, where $i = \sqrt{-1}$, then what is the value of $\frac{z^3+z}{z^2}$?

(a) i (b) 0 (c) 1 (d) –i

Sol. **(b)**

$z = \frac{1+i}{1-i} = i$

$\frac{z^3+z}{z^2} = \frac{-i+i}{-1} = 0.$

23. If f(x), g(x) and h(x) are three polynomials of degree 2 and $\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ then $\Delta(x)$ is a polynomial of degree (dashes denote the differentiation)

(a) 2 (b) 3 (c) 0 (d) at most 3

Sol. **(c)**

$$\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$$

$$\Delta'(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ f''(x) & g''(x) & h''(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow$ $\Delta(x)$ = const

$\Rightarrow$ Degree of $\Delta(x) = 0$

24. If α, β are the roots of $x^2 + px - q = 0$ and γ, δ are the roots of $x^2 - px + q = 0$, then what is $(\beta + \gamma)(\beta + \delta)$ equal to ?

(a) 0 (b) p + q (c) 2q (d) p – q

Sol. **(c)**

α, β roots of $x^2 + px - q = 0$

$\Rightarrow$ $\beta^2 + p\beta - q = 0$ ...(1)

and $\alpha^2 + p\alpha - q = 0$ ...(2)

also γ, δ are roots of $x^2 - px + q = 0$

$\gamma + \delta = p$, $\gamma\delta = q$

$(\beta + \gamma)(\beta + \delta) = \beta^2 + (\gamma + \delta)\beta + \gamma\delta$

$= \beta^2 + p\beta + q$

$= q + q$ (by (1))

$= 2q.$

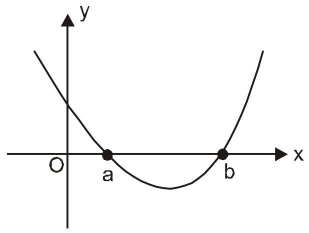
25. The graph of the quadratic polynomial f(x) = (x – a) (x – b) where a, b > 0 and a ≠ b, then the graph does not pass through

(a) first quadrant (b) second quadrant

(c) third quadrant (d) fourth quadrant

Sol. **(c)**

The graph of $f(x) = (x - a)(x - b)$ where $a, b > 0$ and $a \neq b$ is



Not passes through.
third quadrant.

26. There are two sequences whose n^{th} term is given by

1. $a_n = a_{n-1} + a_{n-2}$ where $n > 2$ with $a_1 = 1$, $a_2 = 2$.
2. $a_n = 2a_{n-1}$ with $a_1 = 2$, where $n > 1$.

Which one of the following is correct in respect of the above ?

(a) 1 and 2 are both APs (b) 1 only is in AP
(c) 2 only is in AP (d) Neither 1 nor 2 is in AP

Sol. **(d)**

for Statement –1
$a_n = a_{n-1} + a_{n-2}$, $n > 2$
also $a_1 = 1$, $a_2 = 2$ $\Rightarrow$ 1, 2, 3, 5, 8, ...(1)
for Statement – 2
also $a_n = 2a_{n-1}$; $a_1 = 2$, $n > 1$
Sequence 2 is 2, 4, 8, 16, ...(2)

27. The product of n positive different real number is 1. The sum of these n real numbers

(a) must be an integer (b) less than n
(c) n(n + 1) (d) never less than n

Sol. **(d)**

let $a_1, a_2, a_3, \ldots, a_n$ are n positive and distinct real numbers given $a_1 a_2 a_3 \ldots a_n = 1$
AM > GM
$\Rightarrow$ $a_1 + a_2 + a_3 + \ldots + a_n > n(a_1 a_2 a_3 \ldots a_n)^{1/n} = n$.

28. What is the sum of the infinite series $1 + \frac{2 \times 9}{10} + \frac{3 \times 9^2}{10^2} + \frac{4 \times 9^3}{10^3} + \ldots$?

(a) 10 (b) 100 (c) 1/100 (d) Does not exists

Sol. **(b)**

Let $\frac{9}{10} = x$ then sum of series is $S = 1 + 2x + 3x^2 + 4x^3 + \ldots = \frac{1}{(1-x)^2}$

$= \frac{1}{\left(-\frac{9}{10}\right)^2} = 100.$

29. If A is a non-singular square matrix such that $A^2 - A + I = 0$, then, what is A^{-1} equal to ? (I is identity matrix of same order as A)

(a) I (b) I + A (c) A^2 (d) I – A

Sol. **(d)**

$A^2 - A + I = 0$
$A - I + A^{-1} = 0$
$A^{-1} = I - A$.

30. A group of order 6 has an element of order

(a) 2 (b) 4 (c) 5 (d) 7

Sol. **(a)**

Order of element is a divisor of order of group.

31. If the surface area of a spherical balloon is increasing at the rate of 16 square cm per second when the radius is 40 cm, at what rate is the volume increasing at that moment ?

(a) 320 cubic cm per second (b) 160 cubic cm per second

(c) 150 cubic cm per second (d) 120 cubic cm per second

Sol. **(a)**

Surface area of balloon is $S = 4\pi r^2$

$$\frac{dx}{dt_{(r=40)}} = 16\ cm^2/sec. = 8\pi r\frac{dr}{dt}.$$

Volume of balloon is $V = \frac{4}{3} r\pi^3$

$$\frac{dv}{dt} = 4\pi r^2\frac{dr}{dt} = \frac{r}{2}\left(8\pi r\frac{dv}{dt}\right)$$

$$\frac{dv}{dt_{(r=40)}} = \frac{40}{2}(16) = 320\ cm^3/sec.$$

32. What is the degree of the differential equation $\frac{d^2y}{dx^2} = y^2\sqrt{y}$?

(a) 1 (b) 2 (c) 3 (d) 4

Sol. **(a)**

$$\frac{d^2y}{dx^2} = y^2\sqrt{y}$$

so degree is (1).

33. The general solution of the differential equation $y\frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2 x + 1$ is $Ay - A^2y - A^2x = 1$, where A is the arbitrary constant ($A \neq 0$). Which one of the following solutions of the given differential equation is not a particular solution of the equation ?

(a) $y - 4x = 1$ (b) $3y - 9x = 1$ (c) $y + 2x = 0$ (d) $y^2 = 4x$

Sol. **(d)**

$\because \quad Ay - A^2x = 1$

$\Rightarrow \quad A\frac{dy}{dx} - A^2 = 0 \quad \Rightarrow \quad \frac{dy}{dx} = A$

$\Rightarrow \quad y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 1$ by (2) $\quad \Rightarrow \quad y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + 1$ which is solution of given equation.

$\Rightarrow \quad \frac{dy}{dx} = A$

$\Rightarrow \quad y = Ax + B.$ (Linear equation)

$\Rightarrow \quad y^2 = 4x$ is not a particular solution.

34. What is the solution of the differential equation $\frac{dy}{dx} = \frac{y}{2y\left(\sin y^2 + \cos y^2\right) - x}$?

(a) $xy + \cos y^2 - \sin y^2 = c$
(b) $xy + \cos 2y = c$
(c) $xy - y^2 = c$
(d) $xy = c$

Sol. **(b)**

Let $y^2 = t \Rightarrow 2y$

$\Rightarrow$ $2y(\sin y^2 + \cos y^2) \frac{dy}{dx} - x\frac{dy}{dx} = y$

$\Rightarrow$ $2y(\sin y^2 + \cos y^2)\, dy = xdy + ydx$

$\Rightarrow$ $\int d(\sin y^2 - \cos y^2) = \int d(xy)$

$\Rightarrow$ $\sin y^2 - \cos y^2 + c = xy$

$\Rightarrow$ $xy + \cos 2y = c.$

35. After eliminating the arbitrary constant α from $y = \cos(x + \alpha) + 3\sin(x + \alpha)$ to get a differential equaiton of minimum order as :

(a) $y - 3\frac{dy}{dx} = 1$
(b) $3y + \frac{dy}{dx} = 5$
(c) $y^3 - \left(\frac{dy}{dx}\right)^3 = 9$
(d) $y^2 + \left(\frac{dy}{dx}\right)^2 = 10$

Sol. **(d)**

$y = \cos(x + \alpha) + 3\sin(x + \alpha)$ ...(i)

$\frac{dy}{dx} = -\sin(x + \alpha) + 3\cos(x + \alpha)$ ...(ii)

by $(i)^2 + (ii)^2$

$\left|\frac{dy}{dx}\right|^2 + y^2 = 10$

So d is correct.

36. What is $\int_{-1}^{1} \frac{x^3}{x^2 + 2|x| + 1} dx$ is equal to ?

(a) $\ell n\, 2$
(b) $2\, \ell n\, 2$
(c) 1
(d) 0

Sol. **(d)**

$$\int_{-1}^{1} \frac{x^3}{x^2 + 2|x| + 1} dx$$

Let $f(x) = \frac{x^3}{x^2 + 2|x| + 1}$

$\Rightarrow$ $f(-x) = -\frac{x^3}{x^2 + 2|x| + 1} = -f(x)$ odd function

$\Rightarrow$ $\int_{-1}^{1} \frac{x^3}{x^2 + 2|x| + 1} dx = 0$

so d is correct

37. What is $\int_{\frac{1}{3}}^{\frac{2}{3}} \frac{\ell n x}{\ell n(x-x^2)} dx$ is equal to ?

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{18}$ (d) $\frac{1}{54}$

Sol. **(b)**

$I = \int_{1/3}^{2/3} \frac{\ell n x}{\ell n\left(x-x^2\right)} dx$

$= \int_{1/3}^{2/3} \frac{\ell n x}{\ell n x + \ell n(1-x)} dx$ ...(i)

use property $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$I = \int_{1/3}^{2/3} \frac{\ell n(1-x)}{\ell n(1-x) + \ell n x} dx$ ...(ii)

Equation (i) + (ii)

$2I = \int_{1/3}^{2/3} dx = \frac{1}{3}$

$I = \frac{1}{6}$

so b is correct

38. What is the derivative of $\log_e e^x$?

(a) 2x (b) e (c) 1 (d) 2e

Sol. **(c)**

Let $y = \log_e e^x = x$

$\frac{dy}{dx} = 1$ so c is correct

39. Let f(x) be a polynomial of degree three satisfying f(0) = –1 and f(1) = 0. Also '0' is a stationary point of f(x) but f(x) has no extremum value at x = 0. What is $\int \frac{f(x)}{x^3-1} dx$ equal to ?

Where c is the constant of integration.

(a) $\left(\frac{x^2}{2}\right) + c$ (b) x + c (c) $\left(\frac{x^2}{3}\right) + c$ (d) None of the above

Sol. **(b)**

Let $f(x) = (x-1)(ax^2 + bx + c)$

$f'(x) = (x-1)(2ax + b) + (ax^2 + bx + c)$

$\therefore f'(0) = 0 \Rightarrow -b + c = 0 \Rightarrow b = c$

$\Rightarrow f(x) = (x-1)(ax^2 + bx + b)$

also f(0) = –1

$\Rightarrow (-1)(b) = -1 \Rightarrow b = 1$

$\Rightarrow f(x) = (x-1)(ax^2 + x + 1)$

also $f''(0) = 0 \quad \Rightarrow \quad a = 1 \quad \Rightarrow f(x) = (x + 1)(x^2 + x + 1) = x^3 - 1$

$$\int \frac{f(x)}{x^3 - 1} dx = \int \frac{x^3 - 1}{x^3 - 1} dx = x + c$$

so b is correct.

Directions : For the next **two (02)** questions that follow :

Given $f(x) = \sin x + \cos x$ and $g(x) = \frac{|x|}{x}$ for $x \neq 0$ and $g(0) = 2$.

40. What is $\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} gof\,(x)\,dx$ equal to ?

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{3\pi}{4}$ (d) π

Sol. (d)

$$\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{|f(x)|}{f(x)} dx = \int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\left|\sin\left(x + \frac{\pi}{4}\right)\right|}{\sin\left(x + \frac{\pi}{4}\right)} dx \qquad \therefore\ t = x + \frac{\pi}{4}$$

$$= \int_0^{\pi} \frac{|\sin t|}{\sin t} dt = \pi$$

41. What is $\int_{\frac{3\pi}{4}}^{\frac{7\pi}{4}} gof\,f(x)\,dx$ equal to ?

(a) 0 (b) $-\frac{\pi}{4}$ (c) $-\frac{3\pi}{4}$ (d) $-\pi$

Sol. (d)

$$\int_{\frac{3\pi}{4}}^{\frac{7\pi}{4}} \frac{\left|\sin\left(x + \frac{\pi}{4}\right)\right|}{\sin\left(x + \frac{\pi}{4}\right)} dx$$

$$= \int_{\pi}^{2\pi} \frac{|\sin t|}{\sin t} dt = -(2\pi - \pi) = -\pi$$

Directions : For the **two (02)** questions that follow :

Let $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and $g(x) = f(x - 1) + f(x + 1)$ for all $x \in R$.

42. What is $\int_{-3}^{0} g(x)\,dx$ equal to ?

(a) 0 (b) 1 (c) 2 (d) None of the above

Sol. **(b)**

$$\int_{-3}^{0} f(x-1)\,dx + \int_{-3}^{0} f(x+1)\,dx$$

t = x – 1 u = x + 1

$$= \int_{-4}^{-1} f(t)\,dt + \int_{-2}^{1} f(x)\,du \qquad = \int_{-4}^{-1} o\,dt + \int_{-2}^{-1} o\,du + \int_{-1}^{1} (1-|u|)$$

$$= 2 - 2\int_{0}^{1} u\,du = 2 - 2 \cdot \frac{1}{2} = 1$$

43. What is $\int_{0}^{3} g(x)\,dx$ equal to ?

(a) 0 (b) 1 (c) 2 (d) None of the above

Sol. **(b)**

$$\int_{0}^{3} f(x-1)\,dx + \int_{0}^{3} f(x+1)\,dx \qquad \therefore u = x + 1$$

$$= \int_{-1}^{2} f(t)\,dt + \int_{1}^{4} f(u)\,du \qquad = \int_{-1}^{1} (1-|t|)\,dt = 2 - 2 \cdot \frac{1}{2} = 1$$

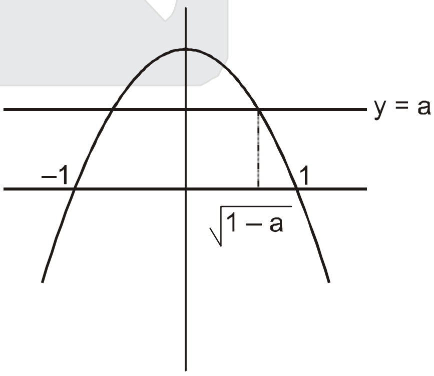
Directions : For the next **two (02)** questions that follow :
Let the area enclosed by the curve $y = 1 - x^2$ and above the line y = a, where $0 \le a < 1$, be represented by A(a).

44. What is $\frac{A(a)}{A(0)}$ equal to ?

(a) $\frac{1}{2}(1 + 2a)\sqrt{1-a}$ (b) $\frac{1}{3}a\sqrt{1-a}$ (c) $\frac{1}{2}(2+a)\sqrt{1-a}$ (d) None of the above

Sol. **(d)**

$$A(a) = 2\int_{0}^{\sqrt{1-a}} (1 - x^2 - a)\,dx$$



$$= 2(1-a)\sqrt{1-a} - \frac{2}{3}\left(\sqrt{1-a}\right)^3$$

$$= 2(1-a)^{3/2} - \frac{2}{3}(1-a)^{3/2} = \frac{4}{3}(1-a)^{3/2}$$

$$A(0) = \frac{4}{3}$$

$$\frac{A(a)}{A(o)} = \frac{\frac{4}{3}(1-a)^{3/2}}{\frac{4}{3}} = (1-a)^{3/2}$$

45. If $\frac{A(0)}{A(1/2)} = k$, then which one of the following is correct ?

(a) k = 0 (b) 0 < k < 0.5 (c) 0.5 < k < 1 (d) None of the above

Sol. (d)

$$\frac{A(o)}{A(1/2)} = \frac{4/3}{4/3\,(1-1/3)^{3/2}} = 2^{3/2}$$

46. The equation of the plane which equally separates the planes 2x – 2y + z + 3 = 0, 4x – 4y + 2z + 5 = 0 is

(a) 8x – 8y + 4z + 13 = 0 (b) 8x – 8y + 4z + 11 = 0

(c) 6x – 6y + 3z + 8 = 0 (d) 6x + 6y + 3z + 8 = 0

Sol. (b)

2x – 2y + z + (3 + 5/2) 1/2 = 0

8x – 8y + 4z + 11 = 0

47. The three planes 2x + 3y – z = 2, 3x + 3y + z = 4, x – y + 2z = 5 intersect

(a) at a point (b) in a line (c) at three points (d) None of the above

Sol. (a)

$$\begin{vmatrix} 2 & 3 & -1 \\ 3 & 3 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 6$$

Directions : For the next **two (02)** questions that follow :

Consider the straight lines $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ and x = y –1 = z.

48. The lines will be perpendicular as well as coplanar if

(a) a = b = c (b) – 2a = b = – 2c (c) 2a = 2b = c (d) a = – 2b = – 2c

Sol. (b)

dot product = 0 a + b + c = 0

and $\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix} = 0$

$\Rightarrow$ a = c, 2a + b = 0

49. The direction cosines of the line $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ (under the condition given in the previous question) are

(a) $\left\langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$ (b) $\left\langle \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$ (c) $\left\langle \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$ (d) None of the above

Sol. (b)

$$\frac{a}{1} = \frac{b}{-2} = \frac{c}{1}$$

d. cs $\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

Directions : For the next **two (02)** questions that follow :

The vectors $x\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+y\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+z\hat{k}$ are coplanar where $x \neq 1$, $y \neq 1$, $z \neq 1$.

50. What is the value of $\frac{1}{1-x}+\frac{1}{1-y}+\frac{1}{1-z}$?

(a) 0 (b) –1 (c) 3 (d) 1

Sol. **(d)**

$$\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1$
$C_3 \to C_3 - C_1$

$$\Rightarrow \begin{vmatrix} x & 1-x & 1-x \\ 1 & y-1 & 0 \\ 1 & 0 & z-1 \end{vmatrix} = 0$$

$\Rightarrow x(y-1)(z-1) - (1-x)(z-1) + (1-x)(0-(y-1)) = 0$
$\Rightarrow x(1-y)(1-z) + (1-x)(1-z) + (1-x)(1-y) = 0$
Divide $(1-x)(1-y)(1-z)$ both side

$$\Rightarrow \frac{x}{1-x}+\frac{1}{1-y}+\frac{1}{1-z} = 0$$

add 1 both side

$$1 + \frac{x}{1-x}+\frac{1}{1-y}+\frac{1}{1-z} = 1$$

$$\frac{1}{1-x}+\frac{1}{1-y}+\frac{1}{1-z} = 1$$

51. What is the value of $xyz - x - y - z$?

(a) 0 (b) 1 (c) 2 (d) –2

Sol. **(d)**

$$\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix} = 0$$

$\Rightarrow x(yz-1) - 1(z-1) + 1(1-y) = 0$
$\Rightarrow xyz - x - z + 1 + 1 - y = 0$
$\Rightarrow xyz - x - y - z = -2$

52. $\vec{a}, \vec{b}, \vec{c}$ are three non-collinear vectors such that $\vec{a}+\vec{b}$ is parallel to $\vec{c}$ and $\vec{a}+\vec{c}$ is parallel to $\vec{b}$. Then which one of the following is correct ?

(a) $\vec{a}+\vec{b}=\vec{c}$ (b) $\vec{b}+\vec{c}=\vec{a}$

(c) $\vec{a}+\vec{c}=\vec{b}$ (d) $\vec{a}, \vec{b}, \vec{c}$ taken in order form the sides of a triangle

Sol. **(d)**

$\vec{a}+\vec{b}=\lambda\vec{c}$ ...(i)

$\vec{a}+\vec{c}=\mu\vec{b}$ ...(ii)

put $\vec{c} = \mu\vec{b} - \vec{a}$ in (i),

we have

$\therefore \quad \vec{a} + \vec{b} = \lambda\left(\mu\vec{b} - \vec{a}\right)$

$(1 + \lambda)\,\vec{a} = (\lambda\mu - 1)\,\vec{b}$

Since $\vec{a}$ and $\vec{b}$ are non collinear vectors so

$1 + \lambda = 0$ & $\lambda x - 1 = 0$

$\lambda = -1$ put $\lambda = -1$

$-\mu - 1 = 0$

$\mu = -1$

$\therefore \ \vec{a} + \vec{b} + \vec{c} = 0$

So d is correct.

Directions : For the next **two (02)** questions that follow :

$\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \dfrac{\vec{b}}{2}$ where $\vec{b}$ and $\vec{c}$ are non-parallel.

53. What angle does $\vec{a}$ make with $\vec{b}$?

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

Sol. (d)

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b}}{2}$$

$$\left(\vec{a}\cdot\vec{c}\right)\vec{b} - \left(\vec{a}\cdot\vec{b}\right)\vec{c} = \frac{\vec{b}}{2}$$

$$\left(\vec{a}\cdot\vec{c} - \frac{1}{2}\right)\vec{b} = \left(\vec{a}\cdot\vec{b}\right)\vec{c}$$

$\because \ \vec{b}$ & $\vec{c}$ are non parallel

so

$\vec{a}\cdot\vec{c} = \dfrac{1}{2}$ & $\vec{a}\cdot\vec{b} = 0$

$\cos\theta = \dfrac{1}{2}$ $\therefore$ $\vec{a} \perp \vec{b}$ So d is correct.

$\theta = \dfrac{\pi}{3}$, where θ is angle between $\vec{a}$ & $\vec{c}$.

So C is correct for Q.54.

54. What angle does $\vec{a}$ make with $\vec{c}$?

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

Sol. (c)

$$\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}}{2}$$

$$\left(\vec{a}\cdot\vec{c}\right)\vec{b} - \left(\vec{a}\cdot\vec{b}\right)\vec{c} = \frac{\vec{b}}{2}$$

comparing

$\vec{a}\cdot\vec{c} = \frac{1}{2}$ and $\vec{a}\cdot\vec{b} = 0$

so $\vec{a} \perp \vec{b}$

$\Rightarrow \theta = 90°$

and $\vec{a}\cdot\vec{c} = \frac{1}{2}$

$|\vec{a}||\vec{c}|\cos\theta = \frac{1}{2}$

$\Rightarrow (1)\ (1)\ \cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

55. If $\vec{a}$ and $\vec{c}$ are perpendicular vectors, then for any vector $\vec{b}$ the vectors $\vec{a}\times(\vec{b}\times\vec{c})$ and $(\vec{a}\times\vec{b})\times\vec{c}$ are

(a) perpendicular (b) parallel

(c) each equal to zero vector (d) parallel respectively to $\vec{a}$ and $\vec{c}$

Sol. (a)

$\vec{a} \perp \vec{b} \quad \Rightarrow \quad \vec{a}\cdot\vec{c} = 0$

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

$= 0 - (\vec{a}\cdot\vec{b})\vec{c} = \lambda\vec{c}$

$(\vec{a}\times\vec{b})\times\vec{c} = -\vec{c}\times(\vec{a}\times\vec{b})$

$= -((\vec{c}\cdot\vec{b})\vec{a} - (\vec{c}\cdot\vec{a})\vec{b})$

$= -(\vec{c}\cdot\vec{b})\vec{a}$

$= \mu\vec{a}$

but $\vec{a} \perp \vec{c}$

so $\vec{a}\times(\vec{b}\times\vec{c})$ and $(\vec{a}\times\vec{b})\times\vec{c}$ are perpendicular.

56. Standard deviation is independent of

(a) change of scale and origin (b) change of scale but not origin

(c) change of origin but not scale (d) neither change of scale nor origin

Sol. (c)

57. X and Y are two related variables. The two regression equation are given by
$2x - y - 20 = 0$ and $2y - x + 4 = 0$.
If $\sigma_y = 1/4$, then what is σ_x equal to ?

(a) 1 (b) 1/2 (c) 1/4 (d) 4

Sol. (b)

$y = 2x - 20$

$\Rightarrow \frac{\sigma_x}{\sigma_y} = 2$

$\Rightarrow \sigma_x = 2 \cdot \sigma_y$

$\Rightarrow \sigma_x = 2 \cdot \frac{1}{4} = \frac{1}{2}$

58. The price of petrol per litre during the four quaters (each quater consists of three months) of a year is Rs. w, Rs. x, Rs y and Rs. z respectively. A person spends Rs. 200 per month on petrol every month. The average price at which he purchases petrol during the year, in rupees is

(a) $\frac{w+x+y+z}{4}$ (b) $\sqrt{\frac{w+x+y+z}{\frac{1}{w}+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}}$ (c) $\sqrt[4]{wxyz}$ (d) $\frac{4wxyz}{wxy+xyz+yzw+zwx}$

Sol. **(d)**
Average = Harmonic mean

$$=\frac{4}{\frac{1}{wx}+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$$

59. Following table summarizes the mean and standard deviation of rainfall in three stations A, B and C observed over a period of one month :

Station	A	B	C
Mean rainfall in mm	32	18	44
Standard deviation	6	8	10

In which station/stations is the rainfall consistent ?
(a) A (b) B (c) C (d) A as well as B

Sol. **(a)**

60. The average weight of 9 mean is x kg. After another man joins the group, the average increases returns to the old level of x kg. Which one of the following is true ?
(a) The 10th and 11th men weight the same
(b) The 10th man weights half as much as the 11th man
(c) The 10th man weights twice as much as the 11th man
(d) None of the above

Sol. **(d)**

$$\frac{w_1+w_2+.......+w_9}{9}=x$$

$$\frac{(w_1+w_2+.......+w_9)+w_{10}}{10}=x=\frac{1}{20}x$$

$$9x+w_{10}=\frac{1}{2}x$$

$$w_{10}=\frac{3}{2}x$$

$$\frac{(w_1+...+w_9)+w_{10}+w_{11}}{11}=x$$

$$9x+\frac{3}{2}x+w_{11}=11x \Rightarrow w_{11}=\frac{x}{2}.$$

61. Consider the following statements in respect of the relation between two integers m and n as m is related to n if (m – n) is divisible by 5.
I. The relation is an equivalence relation.
II. The collection of integers related to 1 and the collection of integers related to 2 are disjoint.
Which of the above statements is/are correct ?
(a) I only (b) II only (c) Both I and II (d) Neither I nor II

Sol. **(c)**

m R n $\Leftrightarrow$ (m – n) is divisible by 5

Reflexive Relation : $\because$ m – m = 0 is divsible by 5

$\Rightarrow$ m R m

$\Rightarrow$ R is reflexive.

Symmetric Relation :

Let m R n $\Rightarrow$ m – n = 5λ $\quad \lambda \in j$

$\Rightarrow$ n – m = 5(–λ) $\quad -\lambda \in j$

$\Rightarrow$ n R M

$\Rightarrow$ R is symmetric.

Transitive :

Let m R n and n R w

$\Rightarrow$ m – n = 5λ, & ($\lambda_1 + \lambda_2$)

$\Rightarrow$ m – w = 5λ

$\Rightarrow$ m R w

$\Rightarrow$ R is Transitive

$\Rightarrow$ R is equivalence relation.

Collection of integers related to 1 are n_i, $n_i \in I$

$\Rightarrow$ $1 - n_i = 5\lambda$,

$\Rightarrow$ $r_i = 1 - 5\lambda$, $\lambda_I \in I$ ...(1)

Collection of integers related to 2 are m_i

$\Rightarrow$ $m_i = 2 - 5\lambda_2$, $\lambda_2 \in I$ ...(2)

Obviously $n_i \neq m_j$.

62. The values of p, for which the quadratic equation $x^2 - 4px + 4p(p-1) = 0$ possesses roots of opposite sign, lies in

(a) (0, 1) (b) $(-\infty, 0)$ (c) (1, 4) (d) $(1, \infty)$

Sol. **(a)**

$x^2 - 4px + 4p(p-1) = 0$

roots of opposite sign $\Rightarrow$ product < 0

4p (p – 1) < 0

$p \in (0, 1)$

63. Let U be the set of 4-digit numbers that can be formed with the digits 2,3,6,7,8,9, each digit being used only once in each number and V be the set of 4-digit numbers that can be formed with the digits 9,8,7,1,4,5, each digit being used only once in each number.

Consider the following in respect of the above :

1. The number of elements of $U \cup V = n^4 - 280$ for some positive integer n.
2. The number of elements of $U + 280 = m^2/10$ for some positive integer m.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

Sol. **(b)**

No. of elements in U are ${}^6P_4 = 360$

No. of elements in V are ${}^6P_4 = 360$

No. of elements in $U \cup V$ = 360 + 360 = 720

(as U and V are disjoint)

$\Rightarrow$ $720 = n^4 - 280$

$\Rightarrow$ $n^4 = 1000$

$\Rightarrow$ $n \notin I$

Statement – 1 is false.

No. of elements of U + 280 = 360 + 280

$$\Rightarrow \quad 640 = \frac{m^2}{10}$$

$\Rightarrow$ m = 80

Statement – 2 is correct.

64. A five-digit number divisible by 3 is to be formed using the numerals 0,1,2,3,4 and 5 without repetition. The total number of ways in which this can be done is

(a) 216 (b) 240 (c) 600 (d) 3125

Sol. **(a)**

0,1,2,3,4,5 divisible by 3.

using digit 1,2,3,4,5 so number = 5! = 120

using digit 0,1,2,4,5 so number = 4 × 4 × 3 × 2 = 96 = 216

65. The value(s) of λ(real or complex) for which the following system of linear equations $(\lambda - \cos\theta)\, x - (\sin\theta)\, y = 0$ and $(\sin\theta)\, x + (\lambda - \cos\theta)\, y = 0$ admits a solution such that at least one of x,y is different from zero is/are

(a) $\cos\theta \pm i \sin\theta$ (b) $\cos 2\theta \pm i \sin 2\theta$ (c) 0 (d) 1

where $i = \sqrt{-1}$.

Sol. **(a)**

$(\lambda - \cos\theta)\, x - (\sin\theta)\, y = 0$

$(\sin\theta)\, x + (\lambda - \cos\theta)\, y = 0$

for infinite solution D = 0

$$\Rightarrow \begin{vmatrix} \lambda - \cos\theta & -\sin\theta \\ \sin\theta & \lambda - \cos\theta \end{vmatrix} = 0$$

$\Rightarrow (\lambda - \cos\theta)^2 + \sin^2\theta = 0$

$\Rightarrow \lambda^2 - 2\lambda\cos\theta + \cos^2\theta + \sin^2\theta = 0$

$\Rightarrow \lambda^2 - 2\lambda\cos\theta + 1 = 0$

$$\Rightarrow \lambda = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$$

$\Rightarrow \lambda = \cos\theta \pm i\sin\theta$

66. Let A and B be two 3 × 3 matrices whose determinants are 2 and 4 respectively. What is the value of det(adj(AB)) ?

(a) 6 (b) 8 (c) 64 (d) 512

Sol. **(c)**

$|A| = 2,\ |B| = 4$

$|\text{adj}(AB)|$

$= |(\text{adj}B)(\text{adj}A)|$

$= |\text{adj}B||\text{adj}\,A|$

$= |B|^{3-1}\, |A|^{3-1}$

$= (4)^2 \times 2^2$

$= 16 \times 4 = 64$

67. Let A be a 3 × 3 matrix such that $A^T = A^{-1}$. Let B be another 3 × 3 matrix whose determinant is 3. What is the determinant of the matrix $A^T B^3 A$?

(a) 3 (b) 6 (c) 9 (d) 27

Sol. **(d)**

$|B| = 3,\ A^T = A^{-1}$

$\Rightarrow AA^T = I \Rightarrow |AA^T| = 1$

Now $|A^T B^3 A|$

$= |A^T A B^3|$

$= |A^T A|\, |B^3|$

$= (1)|B|^3$

$= 3^3 = 27$

68. Which one of the following is correct ?

(a) $6 \cos 20^\circ - 8 \cos^3 20^\circ - 1 = 0$ (b) $8 \sin^3 10^\circ - 6 \sin 10^\circ + 1 = 0$

(c) $8 \cos 20^\circ - 6 \cos^3 20^\circ + 1 = 0$ (d) $6 \sin^3 30^\circ - 8 \sin 10^\circ + 1 = 0$

Sol. **(b)**

$z(y\sin^3 10^\circ - 3\sin 10^\circ) + 1$

$z(-\sin 30^\circ) + 1$

$z\left(-\frac{1}{2}\right) + 1$

$= 0$

69. If x = cos 15º, then which one of the following is correct ?

(a) $4x^2 - 2 + \sqrt{3} = 0$ (b) $8x^2 - 6x + \sqrt{2} = 0$ (c) $8x^2 + 6x - \sqrt{2} = 0$ (d) $16x^2 - 16x^2 + 1 = 0$

Sol. **(d)**

$x = \cos 15^\circ$

$x = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$x = \left(\frac{1}{\sqrt{2}}\right) . \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$x = \frac{\sqrt{3}+1}{2\sqrt{2}} \Rightarrow 2\sqrt{2}\,x = \sqrt{3}+1$

Squaring

$8x^2 = 3 + 1 + 2\sqrt{3}$

$8x^2 = 4 + 2\sqrt{3}$

$4x^2 - 2 = \sqrt{3}$

$16x^4 + 4 - 16x^2 = 3$

$16x^4 - 16x^2 + 1 = 0$

70. If cos (α + β) = 0, then what is cosec (α – β) equal to ?

(a) cos 2β (b) sec 2β only (c) –sec 2β only (d) ± sec 2β

Sol. **(d)**

$\cos(\alpha + \beta) = 0 \Rightarrow \alpha + \beta = \frac{\pi}{2}$ or $\alpha + \beta = \frac{3\pi}{2}$

$\alpha = \frac{\pi}{2} - \beta$, $\alpha = \frac{3\pi}{2} - \beta$

now $\text{cosec}\left(\frac{\pi}{2} - \beta - \beta\right)$ $\text{cosec}\left(\frac{3\pi}{2} - \beta - \beta\right)$

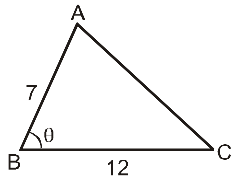
$= \text{cosec}\left(\frac{\pi}{2} - 2\beta\right)$ $= \text{cosec}\left(\frac{3\pi}{2} - 2\beta\right)$

$= \sec 2\beta$ $= -\sec 2\beta$

71. A triangle of area 14 square units has two sides of lengths 7 units and 12 units. What is the angle between the two sides ?

(a) $\sec^{-1}\left(\frac{3}{2\sqrt{2}}\right)$ (b) $\sin^{-1}\left(\frac{2}{3}\right)$ (c) $\tan^{-1}(2\sqrt{2})$ (d) $\text{cosec}^{-1}(2)$

Sol. **(a)**



Area = 14

$\frac{1}{2}(7)(12)\sin\theta = 14$

$\sin\theta = \frac{1}{3}$

$\sec\theta = \frac{3}{2\sqrt{2}}$

$\theta = \sec^{-1}\left(\frac{3}{2\sqrt{2}}\right)$

72. If $\cot^2 x + \operatorname{cosec} x - a = 0$ has at least one solution, then the complete set of values of a belongs to

(a) $[-1, \infty)$ (b) $[-3,-2]$ (c) $(-2,-1)$ (d) None of the above

Sol. **(a)**

$\operatorname{cosec}^2 x + \operatorname{cosec} x - 1 - a = 0$

$(\operatorname{cosec} x + 1/2)^2 = 5/4 + a$

$5/4 + a \geq 9/4$ or $5/4 + a \geq 1/4$

$a \geq 1$ or $a \geq -1$

$a \in [-1, \infty)$

73. If $\tan\alpha = \frac{1+\sqrt{1+\sin 2\theta}}{1-\sqrt{1-\sin 2\theta}}$, where $0 < \theta < \frac{\pi}{4}$, then what is the value of α ?

(a) $\frac{\pi}{2}-\frac{\theta}{2}$ (b) $\frac{\pi}{4}+\frac{\theta}{2}$ (c) $\pi + \theta$ (d) None of the above

Sol. **(a)**

$\tan\alpha = \frac{1+\sqrt{1+\sin 2\theta}}{1-\sqrt{1-\sin 2\theta}}$

$\Rightarrow \tan\alpha = \frac{1+\sqrt{(\sin\theta+\cos\theta)^2}}{1-\sqrt{(\sin\theta-\cos\theta)^2}} = \frac{(1+\cos\theta)+\sin\theta}{(1-\cos\theta)+\sin\theta}$

$= \frac{2\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \cot\frac{\theta}{2}$

$= \tan\left(\frac{\pi}{2}-\frac{\theta}{2}\right) \quad \Rightarrow \alpha = \frac{\pi}{2}-\frac{\theta}{2}$

74. Consider the following statements :

1. For any real number θ, $\sin^6\theta + \cos^6\theta \geq 1$.
2. If x and y are non-zero real numbers such that $x^2 + y^2 = 1$, then $x^4 \geq 1/2$ and $y^4 \geq 1/2$.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

Sol. **(c)**

$\sin^6\theta + \cos^6\theta \leq 1$

$\Rightarrow (\sin^2\theta + \cos^2\theta)^3 - 3\cos^2\theta\sin^2\theta(\sin^2\theta + \cos^2\theta) \leq 1$

$\Rightarrow$ $1 - 3\sin^2\theta \cos^2\theta \leq 1$

$\Rightarrow$ $3\sin^2\theta \cos^2\theta \geq 0$

$\Rightarrow$ $\frac{3}{4}(2\sin\theta \cos\theta)^2 \geq 0$

$\Rightarrow$ $\frac{3}{4}(\sin 2\theta)^2 \geq 0$

Statement 2

$x^4 \leq \frac{1}{2}$ and $y^4 \leq \frac{1}{2}$

$\Rightarrow$ $0 \leq x^2 \leq \frac{1}{\sqrt{2}}$ $0 \leq y^2 \leq \frac{1}{\sqrt{2}}$

Now $x^2 + y^2 \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$\leq \frac{2}{\sqrt{2}}$

$\leq \sqrt{2}$

Both statement are true.

75. Let $0 < \theta < 90°$ be such that $\tan\theta = \sqrt{2}$.

Consider the following statements :

1. There exist distinct integers x,y,z such that $\sec^2\theta = x^2 + y^2 + z^2 - xy - yz - zx$.
2. It is impossible to find integers x,y,z ($x \neq y$, $y \neq z$, $z \neq x$) such that $\tan^2\theta = x^3 + y^3 + z^3 - 3xyz$.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

Sol. **(c)**

$\tan\theta = \sqrt{2}$ $\theta \in (0, 90°)$

Stage-1 $\sec^2\theta = x^2 + y^2 + z^2 - xy - yz - zx$

$\Rightarrow$ $3 = \frac{1}{2}[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$\Rightarrow$ $(x - y)^2 + (y - z)^2 + (z - x)^2 = 6$

If x, y, z are distinct integers then $(x - y)^2$, $(y - z)^2$, $(z - x)^2$ are non integers.

$\Rightarrow$ $(x - y)^2 = 1$, $(y - z)^2 = 1$, $(z - x)^2 = 4$ or combinations of $x - y$, $y - z$ and $z - x$

$\Rightarrow$ $x = 1$, $y = 2$, $z = 3$ and more solutions may be

Statement-1 is correct.

State-2 $\tan^2\theta = x^3 + y^3 + z^3 - 3xyz$

$\Rightarrow$ $x^3 + y^3 + z^3 - 3xyz = 2$

$\Rightarrow$ $(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = 2$

$\Rightarrow$ $x + y + z = 2$ & $x^2 + y^2 + z^2 - xy - yz - zx = 1$

$\Rightarrow$ $x + y + z = 2$ & $(x - y)^2 + (y - z)^2 + (z - x)^2 = 2$

but $x \neq 4$, $y \neq z$ and $z \neq x$

$\Rightarrow$ $(x - y)^2 \geq 1$, $(y - z)^2 \geq 1$, $(z - x)^2 \geq 1$

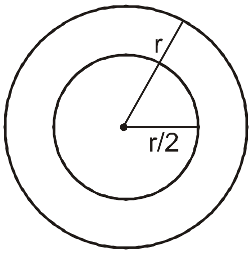
$\Rightarrow$ $(x - y)^2 + (y - z)^2 + (z - x)^2 \geq 3$

Hence x, y, z can not be distinct integer.

76. A point is selected at random from the interior of a circle. The probability that the point is closer to the centre than the boundary of the circle is

(a) 3/4 (b) 1/2 (c) 1/4 (d) 1/8

Sol. (c)



$$\text{Probability} = \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}$$

77. If the letters of the word "ATTEMPT" are written down at random, the probability that all the T's are consecutive is

(a) 1/42 (b) 6/7 (c) 1/7 (d) 1/7

Sol. (c)

ATTEMPT

Total words = $\frac{7!}{3!}$ = 7.6.5.4

= 42 × 20

= 840

All T are together

(TTT), A, E, M, P

words = 5! = 120

Probability = $\frac{120}{840} = \frac{12}{84} = \frac{1}{7}$

78. A football match is played from 4 pm to 6 pm. A boy arrives to see the match (not before the match starts). The probability that he will miss the only goal of the match which takes place at the 15^{th} minute of the match is equal to

(a) 3/4 (b) 1/4 (c) 7/8 (d) 1/8

Sol. (c)

Total 2 hours = 120 minutes

He will miss the only goal of the match which takes place at the 15 minutes of the match so remaining minutes = 105

Probability = $\frac{105}{120} = \frac{7}{8}$

79. Consider the sample space S = {1,2,3,.....} of the experiment of tossing a coin until a head appears. Let n denote the number of times the coin is tossed. A probability space is obtained by setting p(1) = 1/2, p(2) = 1/4, p(3) = 1/8 .....

If an event is defined as A = {n is even}, then what is p(A) equal to ?

(a) 2/3 (b) 2/9 (c) 1/3 (d) 1/9

Sol. (c)

p(1) + p(2) + p(3) + p(4) +........

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...$

$= \frac{1/2}{1-1/2} = 1$

A = {n is even}

p(A) = p(2) + p(4) + p(6) +.......

$= \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + ...$ $= \frac{1/4}{1-1/4} = \frac{1}{3}$

80. If the sum of mean and variance of a Binomial distribution is 4.8 for five trials, then the distribution is

(a) $\left(\frac{1}{3}+\frac{2}{3}\right)^5$ (b) $\left(\frac{1}{4}+\frac{3}{4}\right)^5$ (c) $\left(\frac{1}{5}+\frac{4}{5}\right)^5$ (d) None of the above

Sol. **(c)**

npq + np = 4.8

np(1 + q) = 4.8

5p(2 – p) = 4.8

$5p^2 - 10p + \frac{24}{5} = 0$

$25p^2 - 50p + 24 = 0$

$\Rightarrow \quad 25(p-1)^2 = 1$

$\Rightarrow \quad (p-1)^2 = \frac{1}{25}$

$\Rightarrow \quad p = 1 - \frac{1}{5} = \frac{4}{5}$

$\Rightarrow \quad q = \frac{1}{5}$

81. Nine numbers are successively drawn from the set {1,2,3,4,5,6,7,8,9,10} replacing the number drawn every time before the next draw. Let X denote the number of 1's in the 9 draws and p_r be the probability that X = r(r = 0,1,2,3,...9).

Consider the following statements :

1. $p_0 = p_1$
2. $p_1 = (0.9)^9$

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

Sol. **(c)**

Let p is probability of success $\Rightarrow p = \frac{1}{10}$

$\Rightarrow q = \frac{9}{10}$

$\Rightarrow \quad p_0 = {}^9C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^9 = \left(\frac{9}{10}\right)^9$

$p_1 = {}^9C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^8 = \left(\frac{9}{10}\right)^9 = (0.9)^9$

$\Rightarrow \quad p_0 = p_1$

Statement –1 is correct.

Statement –2 is also correct.

82. A fair coin is tossed a fixed number of times. If the probability of getting seven heads is equal to that of getting nine heads, then the probability of getting two heads is

(a) $\frac{15}{2^8}$ (b) $\frac{2}{15}$ (c) $\frac{15}{2^{13}}$ (d) $\frac{2}{31}$

Sol. **(c)**

Probability of getting head is $p = \frac{1}{2}$

$$q = \frac{1}{2}$$

$\Rightarrow$ $p(7) = p(9)$

$\Rightarrow$ ${}^{n}C_7\, p^7 q^{n-7} = {}^{n}C_9\, p^9 q^{n-9}$

$\Rightarrow$ ${}^{n}C_7\, q^2 = {}^{n}C_9\, p^2$

$\Rightarrow$ ${}^{n}C_7 = {}^{n}C_9$ $(p = q)$

$\Rightarrow$ $n = 16$

$p(2) = {}^{16}C_2\, p^2\, q^{14}$

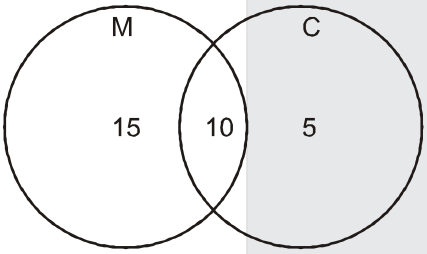
$$= 120.\left(\frac{1}{2}\right)^{16}$$

$$= \frac{15}{2^{13}}.$$

83. In a certain college, 25% of the students failed Mathematics, 15% failed Chemistry and 10% failed both Mathematics and Chemistry. A student is selected at random. If the student failed Chemistry, what is the probability that the student failed Mathematics also ?

(a) 2/9 (b) 2/3 (c) 1/3 (d) 1/9

Sol. (b)



$$P(M \mid c\ |) = \frac{P(m \cap c)}{p(c)} = \frac{10}{15} = \frac{2}{3}$$

84. From a group of 10 persons consisting of 5 lawyers, 3 doctors and 2 engineers, four persons are selected at random. The probability that the selection contains at least one of each category is

(a) 1/2 (b) 1/3 (c) 2/3 (d) 2/9

Sol. **(a)**

Total ways = ${}^{10}C_4$

Favourable case = at least one of each category

$= {}^{5}C_2 \times {}^{3}C_1 \times {}^{2}C_1 + {}^{5}C_1 \times {}^{3}C_2 \times {}^{2}C_1 + {}^{5}C_1 \times {}^{3}C_1 \times {}^{2}C_2$

$= 10 \times 3 \times 2 + 5 \times 3 \times 2 + 5 \times 3 \times 1$

$= 60 + 30 + 15$

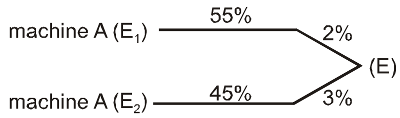
$= 90 + 15 = 105$

$$\text{probability} = \frac{105}{{}^{10}C_4} = \frac{105}{\frac{10.9.8.7}{4.3.2.1}} = \frac{105}{210} = \frac{1}{2}$$

85. In a lot of screws, 55% are produced by machine A and the rest by machine B. Machine A produces 2% defective screws and machine B produces 3% defective screws. The probability that a defective screw randomly found from the lot is manufactured by machine B is

(a) 3/5 (b) 29/51 (c) 14/25 (d) 27/49

Sol. **(d)**



$p(E_1) = \frac{55}{100}$, $p(E/E_1) = \frac{2}{100}$

$p(E_2) = \frac{45}{100}$, $p(E/E_2) = \frac{3}{100}$

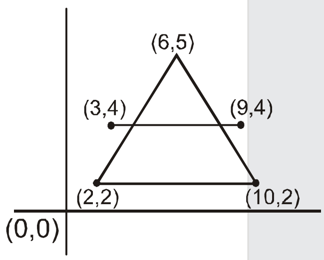
Required probability = $\dfrac{\frac{45}{100}\times\frac{3}{100}}{\frac{45}{100}\times\frac{3}{100}+\frac{55}{100}\times\frac{2}{100}}$

$= \dfrac{45\times 3}{45\times 3+55\times 2} = \dfrac{135}{245} = \dfrac{27}{49}$

86. Suppose the figure 'A' is obtained by joining the points (2,2), (6,5), (10, 2). The horizontal line segment has end points (3,4) and (x, y). Which one of the following is correct ?

(a) x = 7 and y = 4 (b) x = 3 and y = 4 (c) x = 9 and y = 4 (d) x = 6 and y = 4

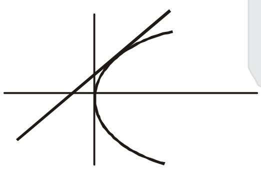
Sol. **(c)**



87. What is the angle made by the tangent to the parabola $y^2 = 4x$ at the point $\left(\frac{1}{4}, 1\right)$ with the y-axis ?

(a) $\tan^{-1}2$ (b) 45º (c) $\cot^{-1}2$ (d) 60º

Sol. **(c)**



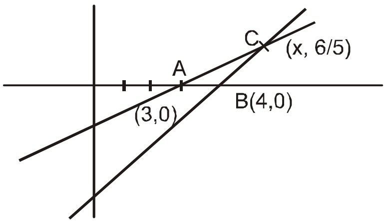
Tangent at $\left(\frac{1}{4},1\right)$ is $y = 2\left(x+\frac{1}{4}\right)$

$\therefore$ Angle made by the tangent with the y-axis is = $\pi/2 - \tan^{-1}2 = \cot^{-1}2$

88. What is the area of the triangle formed by the lines $\frac{x}{3}-\frac{y}{2}=1$, $\frac{x}{4}-\frac{y}{6}=1$ and the x-axis ?

(a) 1/8 square units (b) 1/4 square units (c) 1/5 square units (d) 3/5 square units

Sol. **(d)**



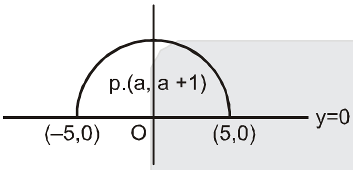
Area of ΔABC is $= \frac{1}{2} \times$ base $\times$ height

$$= \frac{1}{2} \times 1 \times \frac{6}{5} = \frac{3}{5} \text{ square units}$$

89. A region in the xy-plane is bounded by the curve $y = \sqrt{25 - x^2}$ and the line y = 0. If the point (a, a + 1) lies in the interior of the region, then

(a) $a \in (-5, -3)$ (b) $a \in (-\infty, -1) \cup (3, \infty)$
(c) $a \in (3, 5)$ (d) $a \in (-1, 3)$

Sol. **(d)**



$\therefore$ $x^2 + y^2 - 25 < 0$ and $y > 0$

$a^2 + (a + 1)^2 - 25 < 0$ $a + 1 > 0$

$2a^2 + 2a - 24 < 0$ $a > -1$

$a^2 + a - 12 < 0$

$(a + 4)(a + 3) < 0$

$a \in (-4, 3)$ $\therefore$ $a \in (-1, 3)$

90. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $2x - 3y = 0$, then what is the value of c ?

(a) 1 (b) –1 (c) 3 (d) –3

Sol. **(d)**

$6x^2 - xy + 4cy^2 = 0$

$\because$ $m_1 \times m_2 = \frac{a}{b}$ and $m_1 + m_2 = -\frac{2h}{b}$

$\because$ $m_1 = \frac{2}{3}$ $\frac{2}{3} + m_2 = \frac{1}{4c}$

$\therefore$ $\frac{2}{3} m_2 = \frac{6}{4c}$ $\frac{2}{3} + \frac{9}{4c} = \frac{1}{4c}$

$m_2 = \frac{9}{4c}$ $\frac{8}{4c} = -\frac{2}{3}$

$c = -3$

91. If the circles $x^2 + y^2 + 4x + k = 0$ and $x^2 + y^2 + 4y + k = 0$ touch each other, then what is the value of k ?

(a) 1 (b) 2 (c) 4 (d) 18

Sol. (b)

$c_1(-2, 0)$, $c_2(0, -2)$

$r_1 = \sqrt{4-k}$ $\qquad$ $r_2 = \sqrt{4-k}$

$c_1c_2 = r_1 + r_2$

$\sqrt{4+4} = 2\sqrt{4-k}$

$8 = 4(4-k)$

$k = 2$

92. If the point (2, k) is at unit distance from the line $3x - 4y + 1 = 0$, then the values of k are

(a) 1/2 or 3 (b) 1 or 1/2 (c) 1, –1 (d) 0, 1

Sol. (a)

$$\frac{3.2 - 4.k + 1}{\sqrt{3^2 + 4^2}} = \pm 1$$

$7 - 4k = \pm 5$

$4k = 7 \pm 5$

$k = 3, \frac{1}{2}$

93. At the point (0, 7/8), the line joining (2, 1) and (–4, a) is trisected. What is the value of 'a' ?

(a) 13/16 (b) 5/16 (c) 13/8 (d) None of the above

Sol. (d)

2 $\qquad$ 1

(2,1) $\qquad$ (0,7/8) $\qquad$ (–4,a)

$$\frac{-8+2}{3} \neq 0$$

so not possible

1 $\qquad$ 2

(2,1) $\qquad$ (0,7/8) $\qquad$ (–4,a)

$$\frac{-4+4}{3} = 0, \quad \frac{a+2}{3} = \frac{7}{8}$$

$$a = \frac{21}{8} - 2 = \frac{5}{8}$$

94. What is the argument of $-\sqrt{3} - i$?

(a) 30º (b) 150º (c) 210º (d) 240º

Sol. (c)

$$\text{Argument of } (-\sqrt{3} - i) = -\pi + \tan^{-1}\left|\frac{-1}{-\sqrt{3}}\right|$$

$$= -\pi + \frac{\pi}{6} = -\frac{5\pi}{6} = -150^\circ = 210^\circ$$

95. If two straight lines $x\cos\alpha + y\sin\alpha = 1$ and $x\sin\beta + y\cos\beta = 1$ are perpendicular, then which one of the following statements follows logically ?

(a) $\alpha = \beta$ (b) $\alpha - \beta = 90^\circ$ only (c) $|\alpha - \beta| = 90^\circ$ (d) None of the above

Sol. (d)

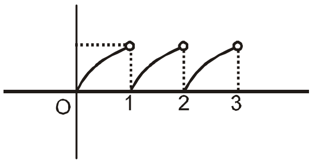
Give lines are perpendicular so $m_1m_2 = -1$

$$\left(-\frac{\cos\alpha}{\sin\alpha}\right)\left(-\frac{\sin\beta}{\cos\beta}\right) = -1$$

$\sin\alpha \cos\beta + \cos\alpha \sin\beta = 0$
$\sin(\alpha + \beta) = 0$
$\alpha + \beta = 0, \pi$

96. Let X be a set of real numbers for which the function $f(x) = \sin(x - [x])$, $x \in R$ is continuous. Then X is equal to
(a) R – Z (b) Z (c) R (d) None of the above
where R is the set of all real numbers and Z is the set of all intergers.
[Here [x] represents greatest integer function]

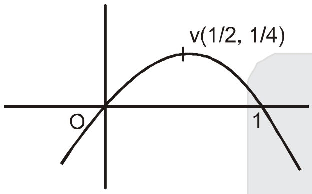
Sol. **(a)**



So for $x \in R - Z$ the function $f(x) = \sin\{x\}$ is continuous.

97. The function $f(x) = x(1 - x)$ defined on (0, 1)
(a) has a minimun value of 1/4
(b) has a maximum value of 1/2
(c) has a maximum value of 1/4
(d) has neither maximum nor minimun.

Sol. **(c)**
$f(x) = x(1 - x)$



So maximum value of f(x) is 1/4.

98. The equation $\sec\theta = \lim\limits_{y \to \alpha x} \dfrac{x^3 - y^3}{3xy^2 + 3yx^2}$ is valid if
(a) α is a negative integer
(b) $\alpha = 0.5$
(c) $\alpha = 1$
(d) $\alpha = 2$

Sol. **(a)**

$$\sec\theta = \lim_{y \to \alpha x} \frac{x^3 - y^3}{3xy^2 + 3yx^2}$$

$$= \frac{x^3(1 - \alpha^3)}{x^3(3\alpha^2 + 3\alpha)}$$

(b) for $\alpha = \dfrac{1}{2}$, $\sec\theta = \dfrac{1 - \frac{1}{8}}{\frac{3}{4} + \frac{3}{2}} = \dfrac{7}{18} < 1$ not possible.

(c) for $\alpha = 1$, $\sec\theta = \dfrac{0}{6} = 0$ not possible.

(d) for $\alpha = 2$, $\sec\theta = \dfrac{-7}{12 + 6} > -1$ not possible.

(a) α is a negative integer

99. For $0 < |x| < \frac{\pi}{2}$, the value of $1 - \frac{\sin x}{x}$ is

(a) strictly positive (b) strictly negative (c) non-negative (d) 0

Sol. **(a)**

$0 < |x| < \frac{\pi}{2}$,

$\because$ $\sin x < x$ for $0 < x < \frac{\pi}{2}$

$\therefore$ $1 - \frac{\sin x}{x}$ is strictly positive

100. If $27a + 9b + 3c + d = 0$, then the equation $4ax^3 + 3bx^2 + 2cx + d = 0$ has at least one real root lying between

(a) -1 and 0 (b) 0 and 3 (c) -3 and -1 (d) None of the above

Sol. **(b)**

Let $f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$f(x) = ax^4 + bx^3 + cx^2 + dx$

$f(0) = 0$

$f(-1) = a - b + c - d$

$f(3) = 3(27a + 9b + 3c + d) = 0$

Since $f(0) = f(3)$ and $f(x)$ is continuous & differentiable function

So $4ax^3 + 3bx^2 + 2cx + d = 0$ has at least one real root lying between 0 and 3.