1. If $A$ and $E$ are non zero and

$$\begin{array}{cccc} A & B & C & D \\ + \; E & F & G & H \\ \hline 2 & 0 & 0 & 9 \end{array}$$

then the value of $C$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

2. The remainder when $10(2009)^4 + 11(2009)^3 + 12(2009)^2 + 13(2009) + 14$ is divided by 2009 is

(A) 0 (B) 12 (C) 14 (D) 1985 (E) 2008

3. What is the sixth decimal digit of $\dfrac{2009^2 + \sqrt{2}}{10^6}$ ?

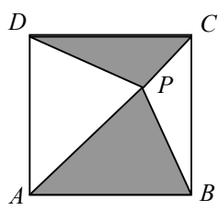
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

4. Kanjana has given Nirmali 12 useful presents. Nirmali has 10 saucepans and they are the only things she has that are made of tin. None of Nirmali's saucepans is of the slightest use. How many tin objects are there among Kanjana's presents?

(A) 0 (B) 2 (C) 10 (D) 12
(E) Cannot be concluded from the given data.

5. $ABCD$ is a square and $P$ is any point inside the square.

The ratio $\dfrac{\text{Area of the shaded region}}{\text{Area of the unshaded region}}$ equals

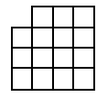
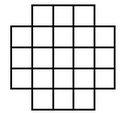
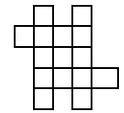



(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{1}$ (D) $\frac{2}{1}$
(E) The exact location of $P$ is required to determine the ratio.



25. A tromino is shown on the right: 

Which of the following can be covered by trominoes without gaps and overlaps?

I. 
II. 
III. 

(A) I only (B) II only (C) I and II only
(D) I and III only (E) All

26. Suppose that 7777 students are participating in the *SLMC 2009*. Suppose also that all of them answer questions (26), (27), (28), (29) and (30) according to the instructions: 'Mark exactly one option for each question or leave it unmarked'. What can you conclude?

I. There can be five students who have answered the five questions in the same way.
II. There is at least one student with the following responses:
(26) - A (27) - B (28) - C (29) - D (30) - E
III. There are at least two students who have answered the five questions in the same way.

(A) I only (B) II only (C) I and II only
(D) I and III only (E) All

27. Now suppose that from Nimal, Kamal, Abdul and Meena some have changed their clans – see problem 24. When they are questioned on one day about the number of *Whites* among them and on the next day about the number of *Reds* among them, each one of them gives the same reply as follows:
Nimal : "Exactly one"
Kamal : "Exactly two"
Abdul : "Exactly three"
Meena : "Exactly four"

How many *Reds* are there among these four?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

28. For any positive integer $n$, let $f(n)$ = {distinct prime divisors of $n$}.
Which of the following is/ are true?

I. For all positive integers $m$, $n$: $f(mn) = f(m) \cup f(n)$
II. $f(m) = f(n) \Rightarrow m = n$
III. If $P$ is the set of all prime numbers and $A \subseteq P$, there is a positive integer $N$ such that $f(N) = A$.

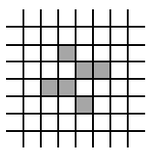
(A) I only (B) II only (C) III only
(D) I and II only (E) I and III only

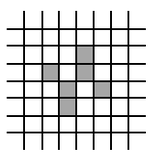
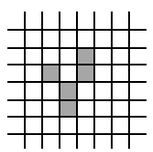
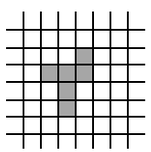
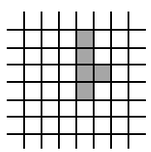
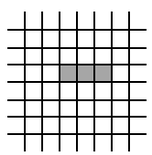
21. A life organism comprising of cells is represented in an infinite draught board by representing a cell by a square. A cell is either alive or dead. All the cells on the board change their state simultaneously according to the following rules to produce the next generation.

(i) A living cell remains alive in the next generation if exactly 2 or 3 of its 8 neighboring cells are alive in this generation, otherwise it dies.

(ii) A dead cell comes to life in the next generation if exactly 3 of its 8 neighboring cells are alive in this generation, otherwise it remains dead.

Find the $2009^{th}$ generation of the life organism whose zeroth generation is shown below. Here the living cells are the shaded squares and the dead cells are the unshaded squares.



(A)  (B)  (C)  (D)  (E) 

22. In how many different ways a bangle can be made by selecting 6 beads from 6 green beads and 6 blue beads?

(A) 7 (B) 8 (C) 12 (D) 13 (E) 14

23. A gang of thieves has discovered that the 5 digit combination to a safe is even and has exactly one odd digit and exactly two digits equal to each other. How many possible combinations are there for them to try?

(A) 1480 (B) 3600 (C) 5400 (D) 7200 (E) 8400

24. In the *Land of Liars*, *White clansmen* always tell the truth, *Red clansmen* always lie and *Black clansmen* alternate between lying on one day and telling the truth the next day. Nimal, Kamal, Abdul and Meena, who belong to either the *White* or the *Red clan*, when interviewed reply as follows:
Nimal : "Kamal is a *Red*"
Kamal : "I am the only *White* among the four of us"
Abdul : "At least one of Nimal and Meena is a *Red*"
Meena : "All of us are *Whites*"

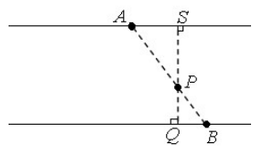
How many *Reds* are there among these four?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4



6. Two cars positioned 800m apart take off at the same time and travel from opposite directions towards each other on parallel straight roads. If the two cars travel at constant speeds of $15ms^{-1}$ and $25ms^{-1}$ respectively, after how many seconds do the cars pass each other?

(A) 40 (B) 35 (C) 30 (D) 25 (E) 20

7. Two cars $A$ and $B$ travel at constant speeds towards each other from opposite directions on parallel straight roads. $A$ travels at $30ms^{-1}$. A lamp post $P$ is situated between the roads such that $QP : PS = 1 : 2$ as shown in the figure. If $B$ travels such that $B$, $P$, $A$ are always in a straight line, what is the speed of $B$?




(A) $15ms^{-1}$ (B) $30ms^{-1}$ (C) $45ms^{-1}$ (D) $60ms^{-1}$ (E) $90ms^{-1}$

8. Let $k$ be the product of any 41 consecutive positive integers. Which of the following is/are always true?

I. 2 divides $k$ II. 3 divides $k$ III. 2009 divides $k$

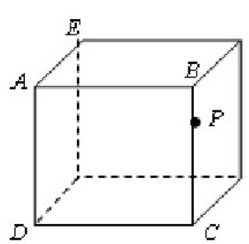
(A) I only (B) II only (C) III only (D) I and II only (E) All

9. Find the value of $x_5$ in the following system of linear equations:

$$x_1 + x_5 + x_9 = 234$$
$$x_4 + x_5 + x_6 = 234$$
$$x_7 + x_5 + x_3 = 234$$
$$x_1 + x_4 + x_7 = 234$$
$$x_9 + x_6 + x_3 = 234$$

(A) 60 (B) 78 (C) 96 (D) 117 (E) 468

10. In the cube shown below, $A, B, C, D, E$ are vertices and $P$ is on $BC$ such that $BP : PC = 1 : 3$. The plane passing through $A, E$ and $P$ divides the cube into two parts. The ratio $\frac{\text{Volume of the smaller part}}{\text{Volume of the larger part}}$ equals




(A) $\frac{1}{8}$ (B) $\frac{1}{7}$ (C) $\frac{1}{6}$ (D) $\frac{1}{5}$ (E) $\frac{1}{4}$

11. The following coded enemy message of a message written in English is intercepted and it contains the word "*ATTACK*". It also contains the word "*NEM*" the name (nom de Guerra name) of a leader at the end of the message. It is also known that different letters in the coded message stand for different English letters in the message. What is the time of the attack?

*DWWDFNDWWHQDPXVHVXLFLGHFDGHUQHP*

(A) 6.00 a.m. (B) 7.00 a.m. (C) 7.00 p.m.
(D) 10.00 a.m. (E) 10.00 p.m.

12. The faces of two fair cubes are painted either red or blue. The first cube has five red faces and one blue face. The probability of the two top faces being of the same color when the cubes are rolled simultaneously is $\frac{1}{2}$. How many red colored faces does the second cube have?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

13. $a_1a_2a_3a_4$ is a four digit positive integer such that
$a_1 + 2a_2 + a_3 = 20$ and $a_2 \times a_4 = 10$.
The minimum value that $a_1 + a_2 + a_3 + a_4$ can take is,
(A) 12 (B) 13 (C) 15 (D) 17 (E) 19

14. A small frog population numbering between 100 and 150 lived in a beautiful pond. One fateful Sunday a monster crane suddenly appeared and feasted on the poor frogs. Thereafter this crane visited the pond every Sunday. On each visit he ate the same number of frogs, and the number of frogs tripled between successive visits. If he ate all the frogs in the pond on his 4th visit, what is the sum of the digits of the number of frogs that were there on his first visit?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

15. In Sudoku, a $9 \times 9$ grid is filled with the integers 1, 2, 3, 4, 5, 6, 7, 8, 9 such that each row, column and indicated $3 \times 3$ grid contains each number only once. What is the sum of the digits of the numbers that should appear in the shaded squares?

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | 7 | 3 | | | |
| 2 | 3 | | | | 8 | | | |
| | | 4 | | | | 1 | 9 | |
| | | 7 | 5 | | | | 1 | |
| | | | | | | | | |
| | 8 | | | | 1 | 9 | | |
| | 5 | 6 | | | | 7 | | |
| | | | 4 | | | | 8 | 2 |
| | | | 6 | 9 | | | | 1 |

(A) 5 (B) 6 (C) 7
(D) 8 (E) 11

16. A sequence $x_1, x_2, \ldots$ is defined as follows:
$x_1 = 1, x_2 = 2, x_3 = 3, x_4 = x_5 = x_6 = 4, x_7 = x_8 = x_9 = 5, x_n = x_{k+1}$
for $n \geq 10$ and $n = 10l + k,\ 0 \leq k < 10$. Find $x_{2009}$.

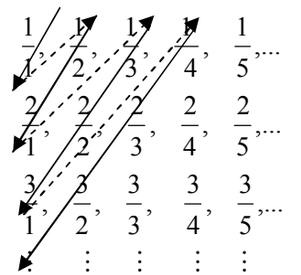
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5



17. $ABC$ is an equilateral triangle and $P$, $Q$, $R$ are points on $AB$, $BC$, $CA$ respectively such that $AP : PB = BQ : QC = CR : RA = 1 : 3$.
Find the ratio $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR}$.

(A) $\frac{2}{1}$ (B) $\frac{8}{3}$ (C) $\frac{16}{7}$ (D) $\frac{4}{3}$ (E) $\frac{8}{5}$

18. Positive rational numbers can be listed with some repetitions in the manner shown on the right by moving along the arrows.



What is the 100th rational number in this list?

(A) $\frac{1}{2}$ (B) $\frac{8}{7}$ (C) $\frac{4}{3}$ (D) $\frac{3}{2}$ (E) $\frac{11}{5}$

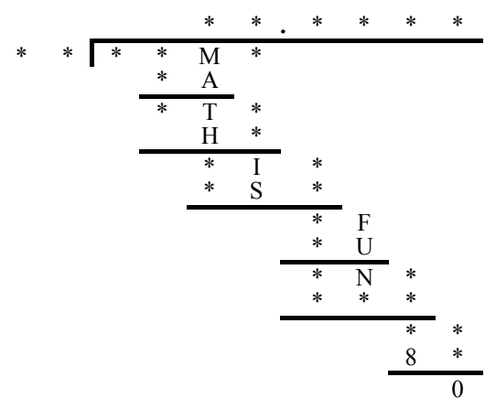
19. Consider the following 'proof':

Step 1: Suppose that the positive integers have a largest integer. Let $n$ be the largest positive integer.
Step 2: Hence $n \geq 1$.
Step 3: But $n \geq n^2$.
Step 4: Therefore $1 \geq n$.
Step 5: So $n \geq 1$ and $n \leq 1$. Therefore $n = 1$. □

What can you conclude?
I. Step 2 is incorrect.
II. Step 3 is incorrect.
III. Step 4 is incorrect.

(A) I only (B) II only
(C) III only (D) I and II only
(E) None of the above.

20. In the long division shown on the right, stars stand for any digit and different letters stand for different digits. The dot in the quotient is the decimal point. Find the divisor.



(A) 10 (B) 20 (C) 16
(D) 40 (E) 80

29. *Infinity Cricket Stadium* built by I. N. Finity is a stadium built like a spiral having seats bearing each positive integer. The $n^{\text{th}}$ seat at this stadium is priced at Rs. $\frac{1}{n}$. During a cricket match when the stadium is full, each person is asked to shift to the next seat, i.e., the person in the $n^{\text{th}}$ seat is asked to shift to the $(n+1)^{\text{th}}$ seat, so that I. N. Finity can have seat number 1. But then the person who was in the $n^{\text{th}}$ seat has to be refunded Rs. $\left(\frac{1}{n}-\frac{1}{n+1}\right)$.

Which of the following is/are true?

I. One Rupee is enough to refund all those who were in seat numbers 1 to $10^{2009}$.

II. Rs. $\left(1-\frac{1}{10^{2009}}\right)$ is not enough to refund all those who were in seat numbers 1 to $10^{2009}$.

III. Rupees $10^{2009}$ is not enough to refund all those who were in seat numbers 1 to $10^{2009}$.

(A) I only (B) II only (C) III only
(D) I and II only (E) II and III only

30. Let $n$ be the number of pairs $(x, y)$ of integer solutions to the following equation:

$$x(x+6) = y^2 + k$$

Which of the following is/are true?

I. $k$ can be given an integer value so that $n = 0$.
II. For any positive integer $m$, $k$ can be given an integer value so that $n = m$.
III. $k$ can be given an integer value so that $n$ is not finite.

(A) I only (B) II only (C) III only
(D) I and III only (E) II and III only.

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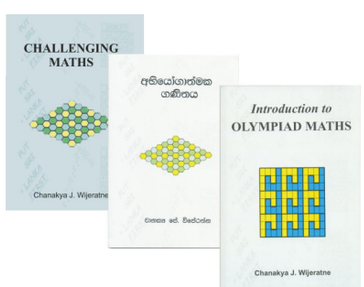
Thank you very much for your participation in the *SLMC 2009*. Your score on this competition will be posted against your index number in www.slmathsolympiad.org. The best 25 students in the *SLMC 2009* will be invited (they will be notified by mail) to participate in the *SLMCC 2009* which will be held on 28th March 2009. In this competition we have tried to showcase mathematics by posing puzzle type questions covering various areas of mathematics. Though the problems require very little knowledge, not more than a Year 10 student's basic mathematics knowledge, some problems might require the mathematical maturity of a student in a higher grade. We hope that this kind of problems will stimulate your mathematical interest beyond classroom mathematics. If you didn't do too well, don't be discouraged! You may have great mathematical talent, but it requires nurturing!! Look for opportunities - there are many websites in the internet and also good books featuring excellent mathematical problems - challenge yourself! For any comments/ suggestions:
e-mail: cjw@maths.cmb.ac.lk, sms or call: 072 3678215




**Index No:** _____

**Medium** ENGLISH

# Sri Lankan Mathematics Competition 2009

## March 07, 2009

This question paper has **30 questions**. The duration of this competition is **90 minutes**. **Answer all questions**. Please read the questions carefully and **fill in the correct lettered circle (only one) against the correct question number in the given answer sheet**. Note that no responses get at least two points while incorrect responses receive zero points. **Please write your index number in the box provided at the top right corner of your question paper**.

**Scoring System for the Sri Lankan Mathematics Competition**

Questions 1 to 10 : 5 points for correct response, 2 points for no response, and 0 points for incorrect response.

Questions 11 to 20 : 6 points for correct response, 2 points for no response, and 0 points for incorrect response.

Questions 21 to 30 : 8 points for correct response, 3 points for no response, and 0 points for incorrect response.

**Sri Lanka Olympiad Mathematics Foundation**